

Analyzing Kepler Exoplanets from their Optical Phase Curves

by

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Do there exist many worlds, or is there but a single world? This is one of the most noble and exalted questions in the study of Nature.

—ALBERTUS MAGNUS

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Contents

1	Introduction	1
1.1	Detection/Characterization of exoplanets	2
1.1.1	Radial velocity method	2
1.1.2	Transit method	4
1.1.3	Transit spectroscopy	7
1.2	Phase curves	8
1.3	Importance of exoplanetary studies	10
2	Phase Curve Model	12
2.1	Generating an orbit	12
2.2	Reflected light	15
2.3	Thermal emission	17
2.4	Ellipsoidal variations	19
2.5	Doppler beaming	20
2.6	Combining the effects	21
3	Fitting	23
3.1	Markov chain Monte Carlo	24
3.1.1	Metropolis-Hastings algorithm	24
3.1.2	Affine-invariant sampler	25

3.1.3	Parallel-tempering algorithm	29
3.2	Formalism	30
3.2.1	Phase one: model comparison	30
3.2.2	Phase two: honing the fits	32
3.3	Synthetic planet tests	32
3.3.1	Planet one	36
3.3.2	Planet two	43
3.3.3	Conclusions	50
4	Kepler K1 Planets	52
4.1	Data reduction	52
4.2	Kepler-13b	53
4.3	HAT-P-7b	62
4.4	Kepler-5b	70
4.5	Kepler-6b and Kepler-8b	78
4.6	Analysis	81
5	Conclusion	84
5.1	Future work	85
5.1.1	Incorporating transits and occultations	85
5.1.2	Non-transiting planets	86
5.1.3	Future targets: K1 and K2	86
	Bibliography	89

Chapter 1

Introduction

The scientific study of exoplanets—planets orbiting stars other than the Sun—is a relatively young field; however, the philosophical concept of there existing other planets in the Universe is actually thousands of years old. As early as the third century BCE, before the development of a rigorous heliocentric model, Epicurus postulated the idea of there being “infinite worlds both like and unlike this world of ours” (quoted in Bailey 1926). Albertus Magnus similarly pondered the existence of other worlds in the thirteenth century (see epigraph; quoted in Bennett et al. 2009). As humanity’s knowledge of the cosmos continued to develop and refine, so too did the treatment of exoplanets. Giordano Bruno, a supporter of heliocentrism, declared “There are countless suns and countless earths all rotating about their suns in exactly the same way as the seven planets of our system,” explicitly postulating the existence of planets about *other* stars (Bruno 1584).

During the nineteenth and twentieth centuries, several “detections” of exoplanets were published, and later retracted. It was only about two decades ago in 1992 that the first widely-accepted discovery of an exoplanet occurred. By analyzing variations in the rotational period of the radio pulsar PSR1257+12, Wolszczan and Frail deduced the presence to two orbiting, Earth-mass bodies (Wolszczan & Frail 1992). Three years later, in 1995, Mayor and Queloz discovered the first exoplanet orbiting a Sun-like star, 51 Peg (Mayor & Queloz 1995). This latter dis-

covery begat an entirely new field of astronomy, involving comprehensive ground- and space-based surveys for exoplanets.

1.1 Detection/Characterization of exoplanets

Several methods have been successfully used to detect and characterize exoplanets, each with its advantages and disadvantages (cf. Seager 2011). In particular, two techniques have dominated the field since the Mayor and Queloz discovery: radial velocity studies and transit observations.

1.1.1 Radial velocity method

The Mayor and Queloz discovery was made using the radial velocity method, which characterizes the so-called “stellar wobble.” In a two-body system, each body exerts a gravitational force on the other, in accordance with Newton’s third law. When one body is considerably more massive than the other, it is often convenient to assume the smaller body (e.g., the planet) orbits the more massive body (e.g., the star). In reality, however, both bodies orbit the system’s barycenter, or center of mass, although the star’s orbit is considerably less pronounced. When the system’s orbital plane is face on (orbital inclination $i = 0^\circ$), the star’s orbit is contained in the plane of the sky, and thus has no radial component along our line of sight. However, if the star’s orbit is inclined ($0^\circ < i \leq 90^\circ$), it will exhibit a radial velocity.

This periodic stellar motion towards and away from us results in a Doppler shift in the light received from the star. Using spectroscopy, we measure these blue- and red-shifts in the star’s spectrum, and derive its radial velocity curve (see Figure 1.1). Using Kepler’s laws of planetary motion and Newton’s law of

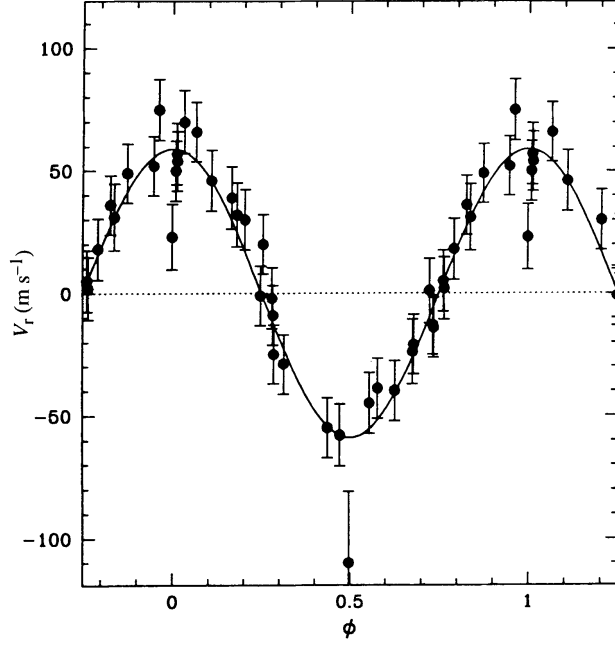


Figure 1.1: Figure from Mayor & Queloz (1995) showing the fluctuations in stellar radial velocity (v_r) of 51 Peg as a function of orbital phase (ϕ) of the exoplanet companion 51 Peg b, which orbits nearly circularly with a period of 4.23 days. By convention, negative and positive v_r indicate motion towards and away from the observer, respectively. The solid black line indicates their model solution to the data (black points).

gravitation, the following equation relates the star's radial velocity and the mass of its planetary companion (Lovis & Fischer 2011):

$$K_{\star} = \sqrt{\frac{G}{1-e^2}} M_p \sin i M_{\star}^{-1/2} a^{-1/2}, \quad (1.1)$$

where the radial velocity semi amplitude $K_{\star}$ is the average of the minimum and maximum radial velocities, G is the gravitational constant, e is the exoplanet's orbital eccentricity, M_p and $M_{\star}$ are the exoplanetary and stellar masses, and a is the semi-major axis of the exoplanetary orbit. Thus, in addition to characterizing the basic shape of an exoplanet's orbit (e and a), the radial velocity technique gives the minimum mass of the planet $M_p \sin i$. Without an independent means of

determining i , radial velocity measurements place a lower limit on the actual mass of the exoplanet. For instance, Mayor and Queloz’s radial velocity analysis gives a minimum mass of $M_p \sin i = 0.47 M_J$. In light of its orbital period ($P = 4.23$ days), 51 Peg b is a hot Jupiter—that is, a planet with mass on the order of Jupiter, in a close-in orbit around its host star.

1.1.2 Transit method

For high orbital inclinations, an exoplanet may pass in front of its host star along our line of sight. This event, called a transit, causes a periodic dimming of the system’s brightness as the exoplanet blocks a portion of the star’s radiant flux (see Figure 1.2). Analysis of the shape, duration, and depth of the transit characterizes several exoplanetary parameters. Assuming the stellar radius is known, the exoplanet’s radius can be determined from δ , the depth of the transit, using

$$\delta = \frac{F_\star - F_{\text{transit}}}{F_\star} = \left(\frac{R_p}{R_\star} \right)^2, \quad (1.2)$$

where F_{transit} and $F_\star$ are the in- and out-of-transit fluxes, and R_p and $R_\star$ are the radii of the planet and star (Seager & Mallén-Ornelas 2003). e , a , and importantly, i are characterized from the duration and shape of the transit (cf. Seager & Mallén-Ornelas 2003; Winn 2011). For certain orbital constructs, the planet may also pass behind the star, in an occultation or secondary eclipse. In the case of both transits and occultations, the parameterizations are better constrained, particularly e , as the eccentricity impacts the temporal separation between transit and occultation.

Clearly then, the combination of radial velocity and transit data allows a full characterization of an exoplanet’s physical dimensions and orbit. For example, HD 209458b was the first planet observed to transit (Charbonneau et al. 2000;

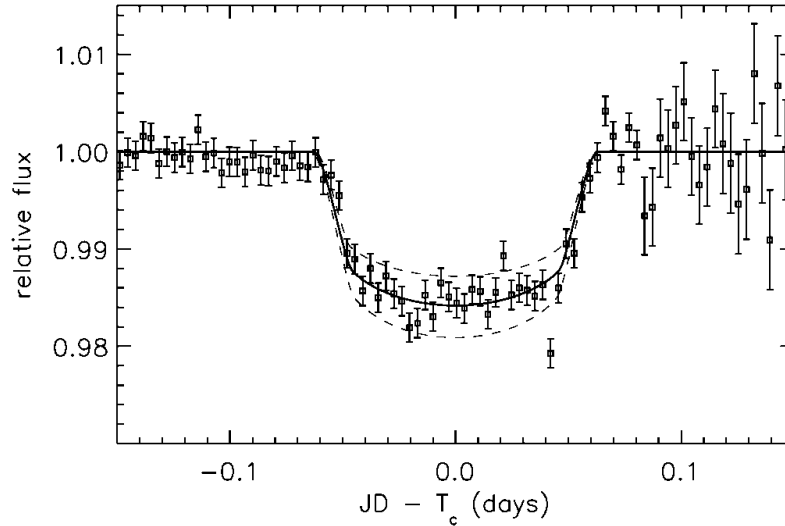


Figure 1.2: Figure from Charbonneau et al. (2000) showing a transit of the hot Jupiter HD 209458b. The solid black line is their best fit solution to the data (black points), while the dashed black lines indicate the effect of increasing and decreasing the planetary radius, namely, causing a deeper and shallower transit, respectively.

see Figure 1.2), but was already identified by radial velocity measurements. Transit analysis indicates a planet with $R_p = 1.27 R_J$ and orbital inclination 87.1° . Including radial velocity analysis yields a true planetary mass of $0.63 M_J$. Armed with both mass and radius, the mean density and surface gravity can be approximated, as well as the escape velocity for different compounds (cf. Charbonneau et al. 2000), which puts very rough constraints on the exoatmospheric composition. Unfortunately, only a small range of inclinations result in an observable transit. Since exoplanetary systems are randomly oriented in the sky, the transiting exoplanet population is a small subset of the general population.

Kepler

The Kepler Space Telescope (hereafter, Kepler) is a space-based observatory dedicated to the discovery of transiting exoplanets, particularly planets orbit-

ing Sun-like stars in their habitable zone—an orbital distance where liquid water exists at temperatures known to support life on Earth. Launched in 2009, Kepler has quasi-continuously observed over 100,000 stars in the same 115.6 deg^2 field of view for just over four years using a 0.95 m Schmidt telescope, until two of its four reaction wheels broke. During this initial phase of the mission, denoted K1, Kepler took either 30 minute long cadence (LC) or 30 second short cadence (SC) integrations with its 42-CCD photometer, each CCD of which measures 2200×1024 pixels. The full Kepler bandpass covers $348 - 970 \text{ nm}$, while the half-maximum bandpass ranges from $435 - 845 \text{ nm}$. Thus, Kepler observes predominantly in the optical and near-infrared spectrum, achieving an impressive photometric precision of 18 ppm over six hours for a 12-magnitude target (Howell et al. 2014). Further technical information can be found in the Kepler Instrument Handbook (Van Cleve & Caldwell 2009) and Kepler Archive Manual (Thompson & Fraquelli 2014).

The K1 phase was rather successful, resulting in over 1,000 confirmed transiting exoplanets and over 3,100 candidates (NASA Exoplanet Archive). Figure 1.3 demonstrates a statistical analysis of Kepler discoveries with periods less than 85 days. It is clear that Kepler succeeded in discovering planets with a wide variety of sizes, ranging from Earths ($0.8 R_{\oplus} - 1.25 R_{\oplus}$) to giant planets ($6 R_{\oplus} - 22 R_{\oplus}$). However, most planets found had sizes of small Neptunes ($2 R_{\oplus} - 4 R_{\oplus}$) and below. These statistics bode well for the relative prevalence of Earth and super-Earth size planets.

Following the loss of the second reaction wheel in 2013, the spacecraft lost its fine-pointing ability, and the Kepler mission began a new phase, called K2. Balancing against photon pressure from the Sun, Kepler now observes fields along the ecliptic plane, taking 30 minute or one minute integrations. Even with the

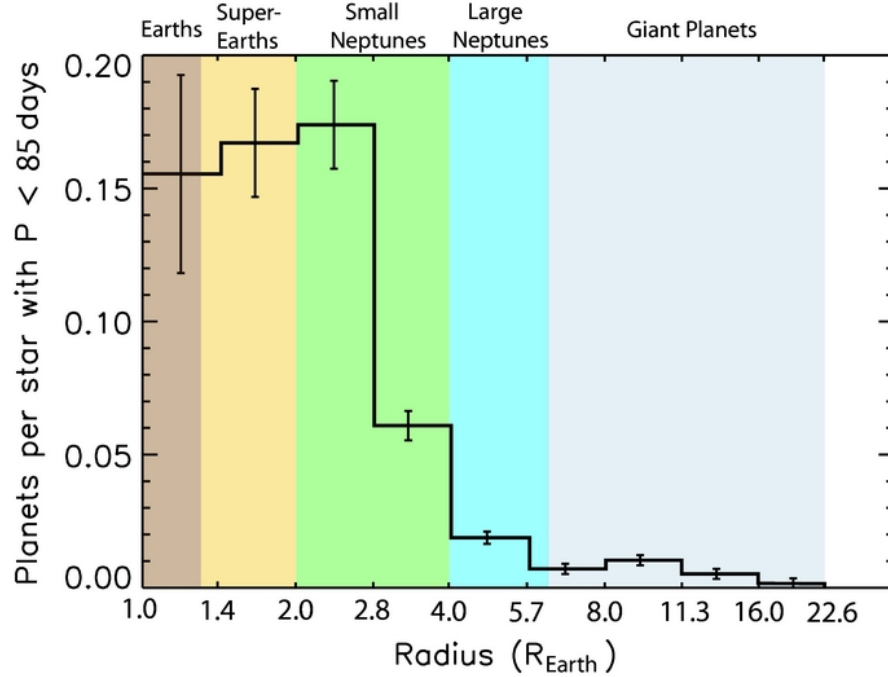


Figure 1.3: Figure from Fressin et al. (2013) showing the average number of planets with periods less than 85 days orbiting mid- to late-type (FGKM) dwarfs, as derived from Kepler discoveries in the first six K1 data sets (quarters). The data has been binned by planet size: Earths ($0.8 R_{\oplus} - 1.25 R_{\oplus}$), super-Earths ($1.25 R_{\oplus} - 2 R_{\oplus}$), small Neptunes ($2 R_{\oplus} - 4 R_{\oplus}$), large Neptunes ($4 R_{\oplus} - 6 R_{\oplus}$), and giant planets ($6 R_{\oplus} - 22 R_{\oplus}$).

loss of its fine-pointing ability, the K2 mission has achieved a six-hour photometric precision of 82 ppm for a 12-magnitude target (Howell et al. 2014). Thus, K2 achieves a precision that is only a factor four worse than that of the original mission. One planet—a super-Earth—has already been confirmed in the K2 data (Vanderburg et al. 2015), and a multi-planet system has recently been announced (Crossfield et al. 2015).

1.1.3 Transit spectroscopy

While broadband, photometric studies of transits indicate a planet’s orbit and physical dimensions, spectroscopically studying planets as they transit probes

their atmospheres (cf. Burrows & Orton 2011; Meadows & Seager 2011). Perhaps the most direct spectroscopic technique is the detection of spectral features in an exoplanet’s irradiated atmosphere. If one is able to characterize the underlying stellar spectrum, either from a model template or ideally from an empirical measurement made during occultation, the planetary atmospheric spectrum is derived by removing the stellar baseline from a spectrum taken during transit.

Analyses of these spectra yield a plethora of information about the planetary atmosphere, for instance, composition, temperature, density, and structure. Recently, rotational broadening of exoatmospheric spectral lines allowed a determination of the rotation velocity of an exoplanet—that is, the length of its day (Snellen et al. 2014). Transit spectroscopy is also the most probable means of detecting extraterrestrial life, by searching for biosignatures—gases indicative of life (cf. Meadows & Seager 2011). Unfortunately, transit spectroscopy requires highly precise spectroscopic data, which prohibits its use as a technique for characterizing exoplanet atmospheres en masse.

1.2 Phase curves

Phase curves are small-scale, variable photometric effects that arise from an exoplanet’s motion about its host star, including the stellar flux reflected and re-radiated by the planet, the beaming effect resulting from the star’s radial velocity, and the ellipsoidal variations due to the planet’s tidal distortion of the star (see Chapter 2 for further discussion). Until the launch of Kepler, the precision required to study these small-scale interactions was not available in a survey context.

Many early phase curve analyses involved measuring the thermal emission from

a planet in the infrared, using facilities such as the Spitzer Space Telescope. For instance, Figure 1.4 shows the increase in infrared emission from transit to occultation of the hot Jupiter HD 189733b as measured by Knutson et al. (2007). Such studies allow rather detailed atmospheric temperature analyses, however, since these instruments are used for research in a wide range of subfields of astronomy, it would be difficult to usurp one for an extended exoplanet survey mission. Kepler’s dedicated exoplanet mission has allowed extended, precise optical observations of exoplanet systems, some of which will have detectable phase curves. Several groups have been working on characterizing individual Kepler exoplanets and brown dwarfs from their phase curves (e.g., Barclay et al. 2012, Placek et al. 2014, Placek & Knuth 2015, Herrero et al. 2014, Shporer et al. 2011). Others have begun to systematically comb through the Kepler K1 database of confirmed exoplanets, characterizing all those they determine to exhibit phase curves (Esteves et al. 2013).

A distinct advantage of phase curve analysis is its ability to fully characterize the physical and orbital parameters of an exoplanet, in addition to rudimentary atmospheric and surface properties. When all four effects can be modeled and fit, the planetary mass is directly calculable without an inclination degeneracy. Additionally, parameters quantifying the reflectivity and temperature on the planet can be derived, which hint at the composition and dynamics of the atmosphere. Such atmospheric information would normally require transit spectroscopy. Thus, phase curves convey a degree of information that normally requires at least two separate techniques, one involving spectroscopy.

Phase curve studies also have the potential to study non-transiting exoplanets. Like the radial velocity technique, the magnitude of the phase curve effects diminishes with decreasing inclination, and thus analysis is constrained solely by the

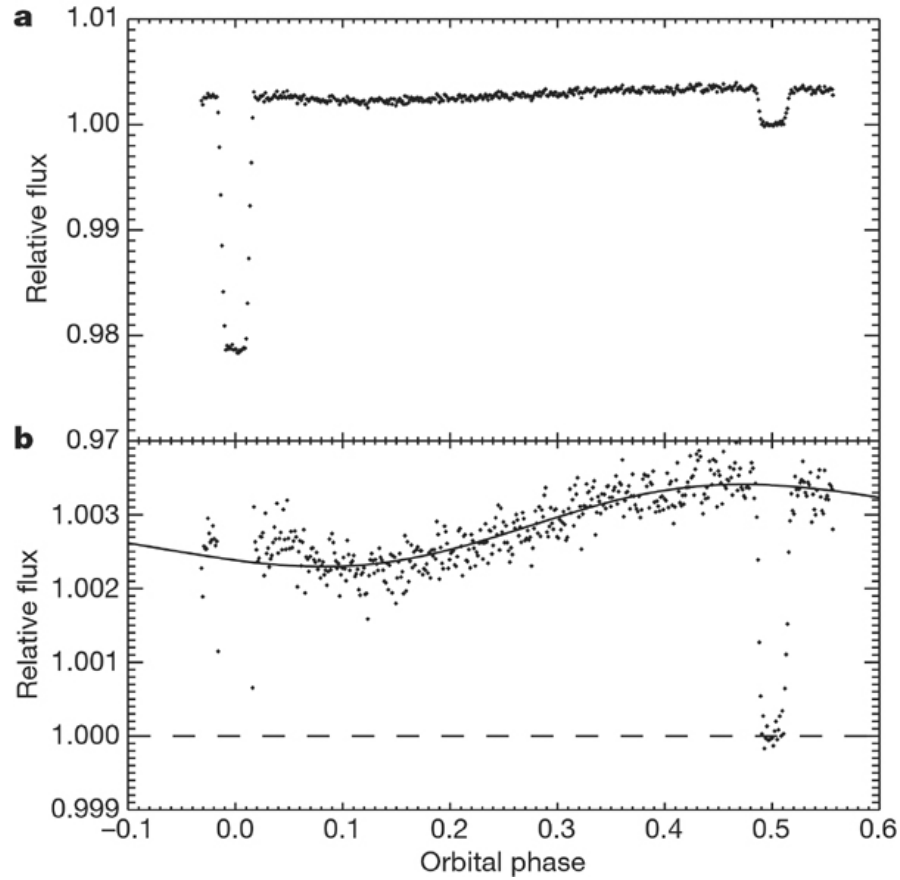


Figure 1.4: Figure from Knutson et al. (2007) showing infrared Spitzer Space Telescope data of the hot Jupiter HD 189733b. The thermal emission curve rises from transit to occultation, indicating the dayside temperature is higher than that of the nightside.

sensitivity of the photometric detector. This means that phase curve studies can fully characterize exoplanets over a greater range of inclinations, allowing studies of the majority of exoplanets that have yet to be fully characterized.

1.3 Importance of exoplanetary studies

Exoplanet detection and characterization is significant for several reasons. Foremost among these is its impact on our understanding of the nature and formation of planetary systems. Simply put, the greater the sample of known planets, the better the conclusions we can draw about their common properties and for-

mation mechanisms. Until the discovery of exoplanets, the scientific community had a pool of only eight major planets and several minor bodies endemic to one solar system with which to study and theorize. With thousands of new worlds to consider, we are able to statistically analyze the overall planet population, which refines our understanding of planetary science.

Exoplanet studies also touch upon other scientific fields, namely chemistry and biology. As previously discussed, transit spectroscopy enables the study of chemical (and potentially biological) processes in disparate environments. Indeed, the detection of biosignatures is at the forefront of astrobiology. Naturally, a priority in the field of exoplanets is the discovery of another Earth-like planet and the existence of extraterrestrial life. Phase curve studies have the potential to significantly broaden the pool of known planets, thereby advancing the field generally.

In Chapter 2, we present the phase curve model we use to analyze light curves. Chapter 3 details our fitting formalism. Our results and analysis for known, transiting exoplanets are set forth in Chapter 4. We conclude in Chapter 5 with a review of our findings, as well as plans for future work.

Chapter 2

Phase Curve Model

Our model of planet phase curves consists of four photometric effects. Two of these—reflected light ($F_{\text{refl}}(t)$) and thermal emission ($F_{\text{thermal,day}}(t)$)—result from planetary contributions to the system’s light curve. Ellipsoidal variations ($F_{\text{ellip}}(t)$) and Doppler beaming ($F_{\text{beam}}(t)$) arise from modulations of the stellar flux via interactions with the orbiting planet. We construct our phase model by adding the individual effects as follows:

$$f(t) = \frac{F_{\text{refl}}(t)}{F_{\star}} + \frac{F_{\text{thermal,day}}(t)}{F_{\star}} + \frac{F_{\text{ellip}}(t)}{F_{\star}} + \frac{F_{\text{beam}}(t)}{F_{\star}} + f_0, \quad (2.1)$$

where $f(t)$ is the normalized light curve flux as a function of time, $F_{\star}$ is the median stellar flux, and f_0 is a variable flux offset. Our model includes neither transits nor occultations. The following sections discuss each component of our model.

2.1 Generating an orbit

Since the four photometric effects we model vary with orbital phase, we first generate an orbit from the input parameters (Table 2.1). We begin by calculating the mean anomaly ($M(t)$), an angular measure of the fraction of the orbital period completed:

$$M(t) = M_0 + \frac{2\pi t}{P}. \quad (2.2)$$

From the mean anomaly, we calculate the eccentric anomaly ($E(t)$), given by

$$E(t) = M(t) + e \sin E(t). \quad (2.3)$$

As shown in Figure 2.1, the eccentric anomaly is the angle between the orbital ellipse’s major axis and the vector from the center of the ellipse to the planet’s projected location on the circumscribed circle.

Orbital parameters	
P	orbital period
a	semi-major axis
e	eccentricity
i	inclination
ω	argument of periastron
M_0	initial mean anomaly
Planetary parameters	
M_p	planetary mass
R_p	planetary radius
A_g	geometric albedo
T_{day}	planetary dayside temperature
Stellar parameters	
$M_\star$	stellar mass
$R_\star$	stellar radius
T_{eff}	stellar effective temperature

Table 2.1: Model input parameters.

Equation 2.3 is transcendental in E , and therefore must be solved numerically. We implement an iterative series solution as outlined in Murray & Dermott (1999). This involves calculating successive values of E using the equation

$$E_{i+1} = M + e \sin E_i, \quad (2.4)$$

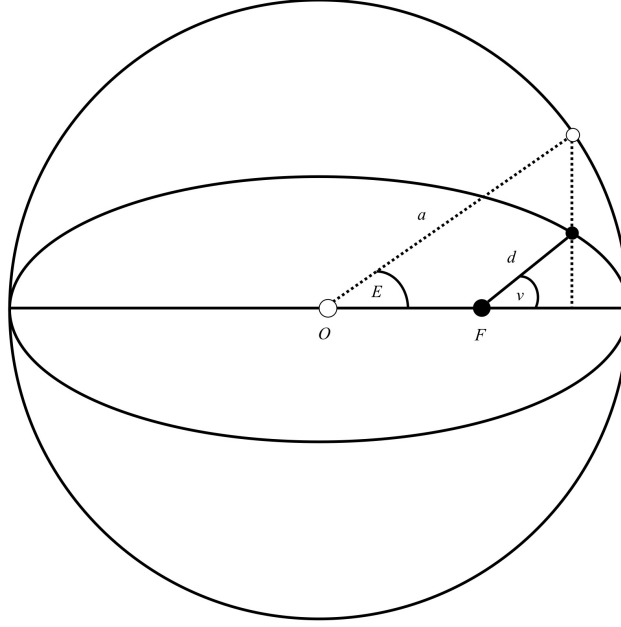


Figure 2.1: Visualization in the orbital plane of the eccentric anomaly E and true anomaly ν . The ellipse represents the planet's true orbital path, centered on the origin O . The star lies at the foci F , a distance d from the planet. The circumscribed circle has radius a equal to the semi-major axis of the orbital ellipse. Adapted from Figure 2.7 in Murray & Dermott (1999).

until a predetermined tolerance for $|E_{i+1} - E_i|$ is achieved. For our model, the tolerance value is 10^{-8} , and per Murray & Dermott (1999)'s suggestion, we initialize our iteration routine with $E_0 = M$. This numerical scheme fails for $e \gtrsim 0.66$ (Murray & Dermott 1999), which informs our choice of bounds for light curve fitting (see Chapters 3 and 4), but should not impact our results significantly as phase curve analysis is most sensitive to large, close-in planets, which tend to have eccentricities close to zero.

Using the eccentric anomaly, we calculate the true anomaly

$$\nu(t) = 2 \arctan \left(\sqrt{\frac{1+e}{1-e}} \tan \frac{E(t)}{2} \right), \quad (2.5)$$

and the planet-star separation

$$d(t) = a[1 - e \cos E(t)]. \quad (2.6)$$

The true anomaly is similar to the eccentric anomaly, except one of the foci of the ellipse—rather than its center—serves as vertex of the angle (see Figure 2.1). For a more detailed treatment of orbital dynamics, and derivations of the above equations, please see Murray & Dermott (1999).

Finally, we calculate the phase angle

$$\theta(t) = \arccos(\sin[\omega + \nu(t)] \sin i), \quad (2.7)$$

which represents the angle between the line-of-sight vector and the star-to-planet vector (Mislis et al. 2012; Placek et al. 2014). The phase angle thereby provides a convenient angular quantification of the planet’s position in its orbit about its host star.

2.2 Reflected light

As a star irradiates an orbiting planet, some of the light reflects off the planet’s surface and atmosphere along our line of sight, thereby contributing to the system’s light curve. Treating the star as an isotropic radiator and the planet as a Lambertian sphere¹, the normalized, reflected flux is given by

$$\frac{F_{\text{refl}}(t)}{F_{\star}} = \frac{A_g}{2} \frac{R_p^2}{[d(t)]^2} [1 + \cos \theta(t)], \quad (2.8)$$

¹A Lambertian sphere reflects incident radiation such that the intensity is independent of viewing angle (Pedrotti & Pedrotti 1987).

where A_g is the geometric albedo (Mislis et al. 2012; Placek et al. 2014). The geometric albedo A_g is the ratio of the planetary reflected flux at full phase to the reflected flux of a Lambertian disk of comparable cross-sectional area (Burrows & Orton 2011). This quantity varies with atmospheric and surface composition, and thus modeling the reflected light and constraining A_g provides a very basic characterization of the planet’s atmosphere. For instance, the presence of clouds serves to increase the geometric albedo (Burrows & Orton 2011).

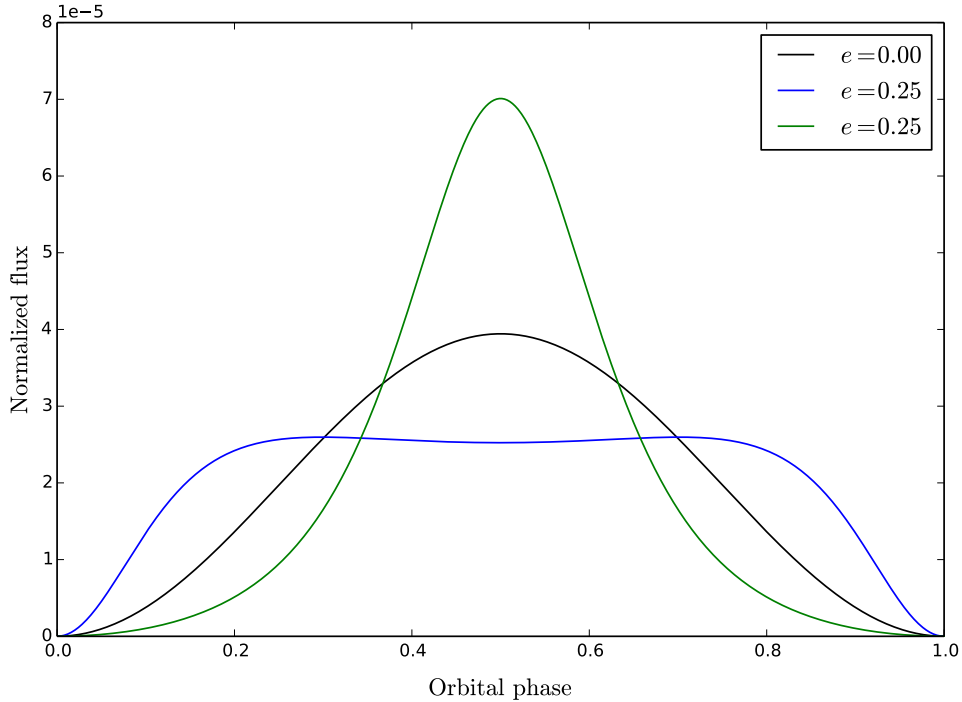


Figure 2.2: Simulated reflected components of three hot Jupiters, differing only in their eccentricities and arguments of periastron. The reflected curves of the circular orbit (black line) and eccentric orbit with periastron on the far side of the star (green line) are both unimodal. The curve of the eccentric orbit with periastron on the near side of the star (blue curve) is bimodal.

For circular and nearly-circular orbits the reflection curve is unimodal (see Figure 2.2, black line). In this case, reflection reaches a maximum at full phase, when the fully-illuminated side of the planet faces us. Conversely, the reflected compo-

ment experiences a minimum at new phase, when the planet’s non-illuminated side faces us. For more eccentric orbits, the reflection curve may remain unimodal (see Figure 2.2, green line) or become bimodal (see Figure 2.2, blue line) depending on the location of periastron/apastron. In the latter case, apastron occurs on the far side of the star, concurrent with full phase. The $[d(t)]^{-2}$ term overcomes the constructive influence of observing a fully-illuminated planetary disk, resulting in a dip in the reflected component. For any eccentricity, however, the overall oscillation of reflected light follows the orbital period.

2.3 Thermal emission

In addition to reflecting its host star’s light, a planet also absorbs and re-radiates stellar flux as thermal emission. Following the example of Placek et al. (2014), we assume the thermal emission of the nightside of the planet is negligible compared to that of the dayside, and therefore only model the latter. Approximating the planet as a blackbody radiator (Placek et al. 2014),

$$\frac{F_{\text{thermal,day}}(t)}{F_{\star}} = \frac{1}{2}[1 + \cos \theta(t)] \left(\frac{R_p}{R_{\star}} \right)^2 \frac{\int B(T_{\text{day}})G(\lambda)d\lambda}{\int B(T_{\text{eff}})G(\lambda)d\lambda}, \quad (2.9)$$

where $B(T)$ is the Planck distribution at temperature T and $G(\lambda)$ is the Kepler response function at wavelength λ (Van Cleve & Caldwell 2009). The equation for thermal emission has a $\theta(t)$ dependence similar to that of reflected light, but lacks the $[d(t)]^{-2}$ factor. Thus, for all eccentricities, thermal emission follows a unimodal pattern with a period corresponding to the planet’s orbit, experiencing maximum emission at full phase and minimum at new phase. The similarity of the reflected and thermal components at low eccentricities introduces a degeneracy to the model. Since the Kepler telescope observes in a single bandpass (348–970 nm),

there is no way to distinguish thermal from reflected photons in such cases. If for higher e the reflected component takes on a bimodal shape, it becomes possible to distinguish the reflection and thermal emission effects (see Figure 2.3, middle panel).

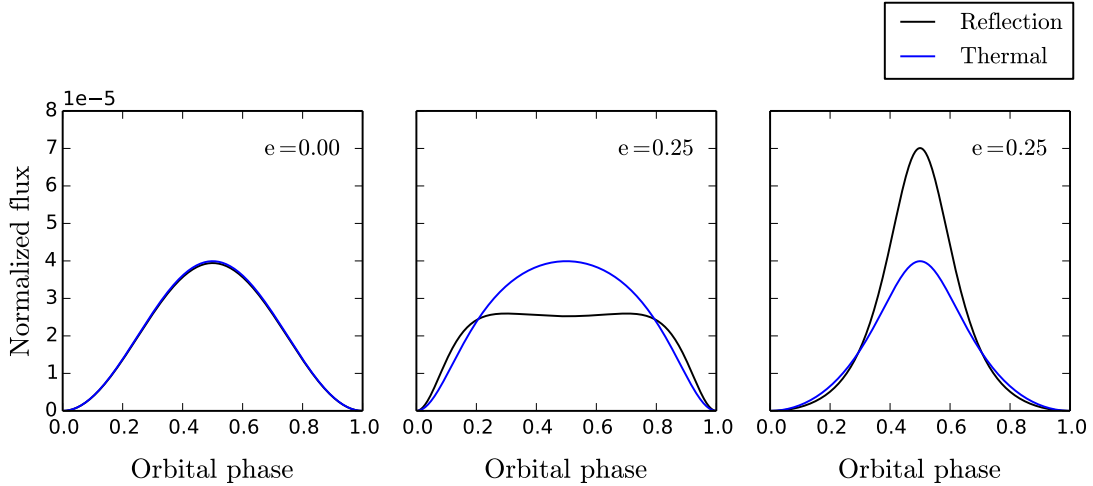


Figure 2.3: Comparison of reflected and thermal emission components of the same three hot Jupiters from Figure 2.2. Note for the circular orbit (left panel), the effects are indistinguishable, while in the case of the reflected component becoming bimodal at higher e (middle panel), the two take different shapes. The right panel demonstrates that if the reflected curve remains unimodal at higher e , the general shapes of the two effects remain relatively similar.

Modeling thermal emission characterizes T_{day} , which gives further insight into the atmospheric properties of the planet. For instance, the relationship between the incident solar flux and T_{day} reveals how efficiently the atmosphere absorbs radiation. If the thermal emission flux exceeds that which the planet receives from its star, the planet likely has an additional, internal energy source, for instance radioactive and/or tidal heating.

2.4 Ellipsoidal variations

Ellipsoidal variations result from the tidal force of the planet on its host star, causing an elongation of the star towards the planet, and thus a periodic oscillation in the stellar flux. Placek et al. (2014) characterize the normalized flux from ellipsoidal variations as

$$\frac{F_{\text{ellip}}(t)}{F_{\star}} = \beta \frac{M_{\text{p}}}{M_{\star}} \left[\frac{R_{\star}}{d(t)} \right]^3 (\cos^2 [\omega + \nu(t)] + \sin^2 [\omega + \nu(t)] \cos^2 i), \quad (2.10)$$

where β is the gravity darkening exponent, given by Mislis et al. (2012) as

$$\beta = \frac{\log_{10}(GM_{\star}/R_{\star}^2)}{\log_{10} T_{\text{eff}}}. \quad (2.11)$$

The constant β accounts for the variation in flux output with surface gravity for a tidally-deformed star. Regions with lower surface gravity will be less radiant than those with high surface gravity (cf. von Zeipel 1924). Unlike reflection and thermal emission, the ellipsoidal variations effect peaks distinctly twice on each orbit, and is thus more easily distinguishable from the other effects. The peaks occur at the quarter phases, when the star presents its greatest cross-sectional area along our line of sight (see Figure 2.5)

The characterization of ellipsoidal variations (as well as Doppler beaming) enables us to constrain the mass of the planet using photometry alone, a feat that previously required Doppler spectroscopy. Along with the radius derived from either transit analysis or the reflection and thermal effects discussed above, the planet's average density is calculable. Thus, with a full phase curve analysis, we can potentially make basic assumptions about the composition of both a planet's

atmosphere and interior.

2.5 Doppler beaming

Doppler beaming results from the reflex motion of a host star about the system’s barycenter—the so-called “stellar wobble.” Special relativity predicts an increase in observed flux from an object moving towards an observer and a decrease in flux for a receding object. Whereas the luminous object may radiate isotropically in its rest frame, in an inertial frame the object’s radiation concentrates along its velocity vector (see Figure 2.4; cf. Rybicki & Lightman 2004). Assuming the star’s radial velocity is non-relativistic and any bandpass effect is negligible, Loeb & Gaudi (2003) give the normalized beaming component as

$$\frac{F_{\text{beam}}(t)}{F_{\star}} = \frac{4v_r}{c}, \quad (2.12)$$

where v_r is the radial velocity of the star—its velocity along our line of sight. Murray & Correia (2011) give the radial velocity for an eccentric orbit as

$$v_r = V_z + K (\cos [\omega + \nu(t)] + e \cos \omega), \quad (2.13)$$

where V_z is the proper motion of the system’s barycenter along our line of sight and K is the radial velocity semi-amplitude. Using the formulation for K from Equation 1.1 and neglecting the proper motion of the system, Equation 2.13 becomes

$$v_r = \left(\frac{2\pi G}{P} \right)^{1/3} \frac{M_p \sin i}{M_{\star}^{2/3} \sqrt{1 - e^2}} (\cos [\omega + \nu(t)] + e \cos \omega), \quad (2.14)$$

where we have assumed the mass and Doppler beaming of the planet is negligible compared to those of the star (Placek et al. 2014).

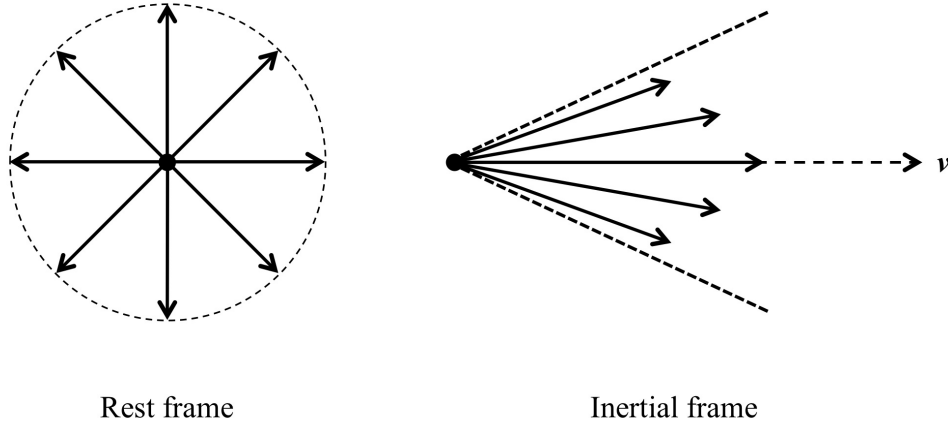


Figure 2.4: A moving object isotropically radiating in its rest frame (left) appears to an observer in an inertial frame to beam its light along its velocity vector (right). Adapted from Figure 4.3 of Rybicki & Lightman (2004).

In our quantification of the four phase effects, Doppler beaming is unique in that it varies between positive and negative relative flux values in a classic sinusoid fashion. Its curve’s unique shape reduces degeneracy between beaming and other effects. Similar to ellipsoidal variations, Doppler beaming places constraints on the planet’s mass, and experiences maxima and minima at quarter phases when the star’s line-of-sight velocity is correspondingly maximized and minimized (see Figure 2.5).

2.6 Combining the effects

When summed, the four phase effects induce unique patterns in the composite phase curve. Consider for instance the three light curves shown in Figure 2.5. As previously discussed, the ellipsoidal variations produce double peaks of equal

height at the quarter phases of the orbit, which can contribute to an overall bimodal feature in the light curve. Reflection and/or thermal components may serve to fill in the trough between peaks, and in cases where these two effects are particularly strong, will generally level off the phase curve near orbital phases of 0.5 (see Figure 3.4, planet one). Since Doppler beaming is the only effect to yield both positive and negative values, including the beaming effect will introduce asymmetries. In the case of Figure 2.5, the beaming causes asymmetric peaks, i.e., peaks of different amplitudes.

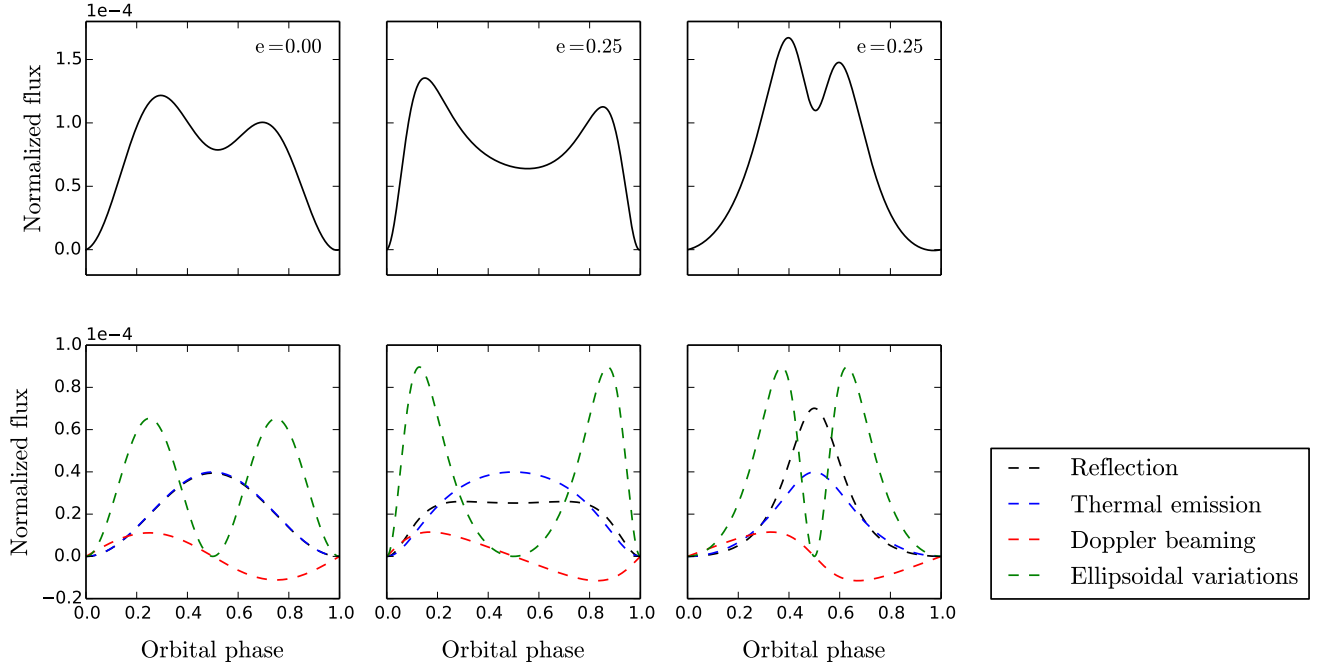


Figure 2.5: Top panel: Combined effects for the three hot Jupiters from Figures 2.2 and 2.3. Bottom panel: Constituent effects.

Chapter 3

Fitting

In choosing a method to fit our phase curve model to photometry, we need to consider several factors. Chief among these is how to cope with parameter degeneracy, which occurs when multiple parameters have a similar effect on the resulting model. For instance, from the equation for thermal emission (Equation 2.9), it is clear that increasing the radius of the planet R_p or increasing the day-side planet temperature T_{day} both serve to increase the amplitude of the thermal contribution to the system’s light curve. In such a case, a fitting routine may have difficulty deducing the correct pair of parameter values. Instead, the routine may return one parameter value inflated and the other depressed, a combination that may give an overall thermal amplitude similar to that of the correct values.

An equally important issue is ensuring the fitting routine actually finds the best fitting set of parameters. Such algorithms aim to minimize the residuals, the absolute difference between a model and data. As with most optimization algorithms, fitting routines often times suffer from the tendency to return a “best-fit” model that actually corresponds to a local minimum in the residuals, rather than the global minimum. Once in a local minimum, many algorithms have a difficult time extracting themselves, since the surrounding residual gradient serves to keep them in the minimum.

3.1 Markov chain Monte Carlo

In an attempt to mitigate these issues, we opt to implement a Markov chain Monte Carlo (MCMC) fitting routine (cf. Ivezić et al. 2014). In general, an MCMC populates a model’s parameter space with one or more walkers, which execute a random walk, evaluating the goodness of fit at each step. Theoretically, the walker(s) will converge to the best-fitting parameters. There are many implementations of the MCMC, varying in computational intensity and implementation of the random walk. In the following subsections, we describe three of these implementations and develop some of the intricacies of MCMC fitting.

3.1.1 Metropolis-Hastings algorithm

The Metropolis-Hastings (MH) implementation is the simplest of the MCMC methods (Foreman-Mackey et al. 2013; Ivezić et al. 2014). Consider an m -dimensional parameter space, containing one walker. Let the m -tuple $X(n)$ denote the walker’s current position at the n th step. To determine the walker’s position at the $n + 1$ step, a proposal position Y is drawn from a proposal distribution in one of the dimensions—for instance, a Gaussian of scale a centered at the walker’s position in the m th dimension. Let $P(X)$ denote the probability distribution, a measure of the goodness of fit at the position X in parameter space. The ratio $\alpha = P(Y)/P(X(n))$ is calculated, and compared to β , a number randomly drawn on the range $[0, 1]$. If $\alpha \geq \beta$, the proposal position Y is accepted such that $X(n + 1) = Y$; otherwise, $X(n + 1) = X(n)$. This process is repeated for n steps.

The setup of even this simplest version of an MCMC has advantages over typical least-squares minimization routines. The random-walk nature of the position

update criterion—which always accepts a better fitting position in addition to a non-negligible fraction of worse-fitting positions—generally enables the MCMC to more completely probe the parameter space, while still achieving greater efficiency than a grid search. The resulting distribution of the walker’s steps in parameter space—the posterior distribution (PD)—can be used to infer error bars for fitted parameters and evaluate degeneracy between parameters (discussed below).

Nonetheless, the MH method tends to perform poorly in the case of degenerate parameters. Since the walker explores parameter space in orthogonal steps—that is, only proposing a change in one dimension per step—its convergence to the best-fitting set of parameters in a non-orthogonal, degenerate subspace will be inefficient. As discussed in the following sections, our phase curve model has several degenerate parameters, and we opt not to implement the MH method of MCMC.

3.1.2 Affine-invariant sampler

The affine-invariant (AI) MCMC (Goodman & Weare 2010; Foreman-Mackey et al. 2013) attempts to resolve parameter degeneracy by using an ensemble of walkers, and proposing steps in multiple dimensions. To this end, the proposal position Y for each walker is selected from a proposal distribution connecting that walker’s current position to that of another randomly selected walker. After an MCMC is well into its run, this proposal selection method will better enable any errant walkers to join walker clumps surrounding a region of high probability. That the walkers make non-orthogonal steps also enables more efficient convergence, as the ensemble can more directly move through non-orthogonal, degenerate subspaces.

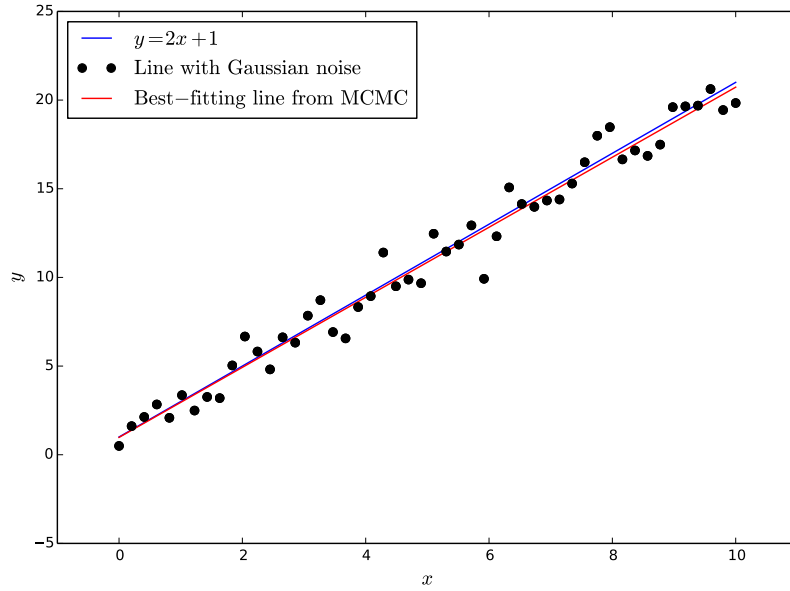


Figure 3.1: Solid blue line: Linear function from which data was derived. Black dots: Data used in MCMC fitting routine, created by adding Gaussian random noise to evenly-sampled points from the linear function. Solid red line: Best-fitting linear function as given by the AI MCMC fitting routine.

To demonstrate the effectiveness of the AI MCMC, consider the process of fitting a linear trend. The data set in Figure 3.1 was created by adding random Gaussian noise to points uniformly sampled from a line. An AI MCMC routine using 100 walkers was run for 500 steps. The results of a run are commonly visualized using step figures and triangle plots. Figure 3.2 is a set of step figures, that is, visualizations of the walkers' movements through each dimension of the parameter space. Each black line corresponds to the path of a single walker. Tracing a line's path through the steps of the run (x -axis) shows the various parameter values occupied by the walker; in this case, the slope m and y -intercept b . The walkers were initially uniformly distributed on the interval $[-5, 5]$ for both m and b . The step figure clearly demonstrates the convergence of the walkers to best fit values from this uniform initial distribution.

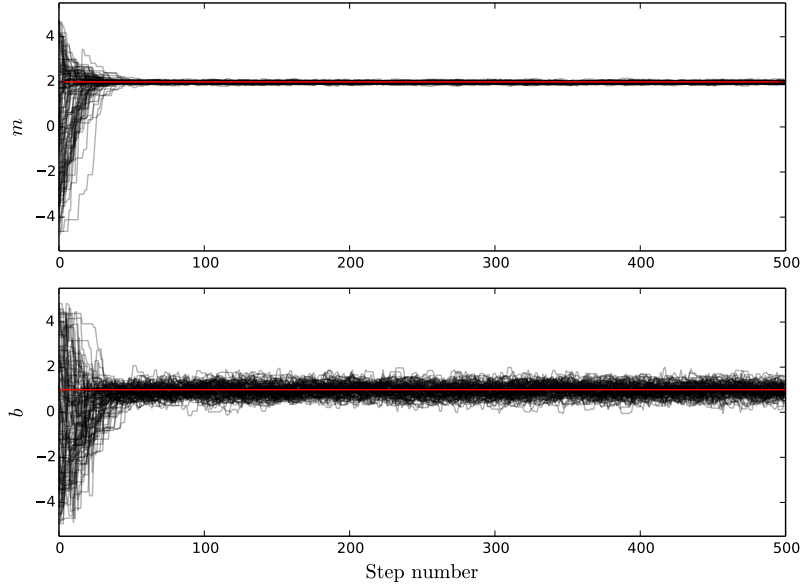


Figure 3.2: Step figures for linear parameters m (top panel) and b (bottom panel), tracing the paths of the AI MCMC walkers through parameter space at each step in the fitting routine. The red overplotted lines indicate the true values $m = 2$ and $b = 1$.

While step figures assist in evaluating the convergence of an MCMC, a triangle plot provides a means to evaluate a fit’s robustness. Figure 3.3 is a triangle plot of the linear fit, with the first 39 steps from each walker discarded as a “burn-in,” so as to remove any influence of the starting distribution. The length of this burn-in is chosen as three times the maximum autocorrelation time, a measure of the number of steps necessary to achieve independent sampling of the PD (Foreman-Mackey et al. 2013). The histograms display the PD for each parameter, all the positions occupied by the walkers following the burn-in phase. For constrained parameters, these one-dimensional PDs should take on a semi-Gaussian shape. Overlaid on the histograms are dashed lines that correspond to the 16th, 50th, and 84th percentiles, effectively showing the median value with 1σ errors. The solid blue line corresponds to the set of best-fitting parameters found by the routine. This ideally corresponds to the median value.

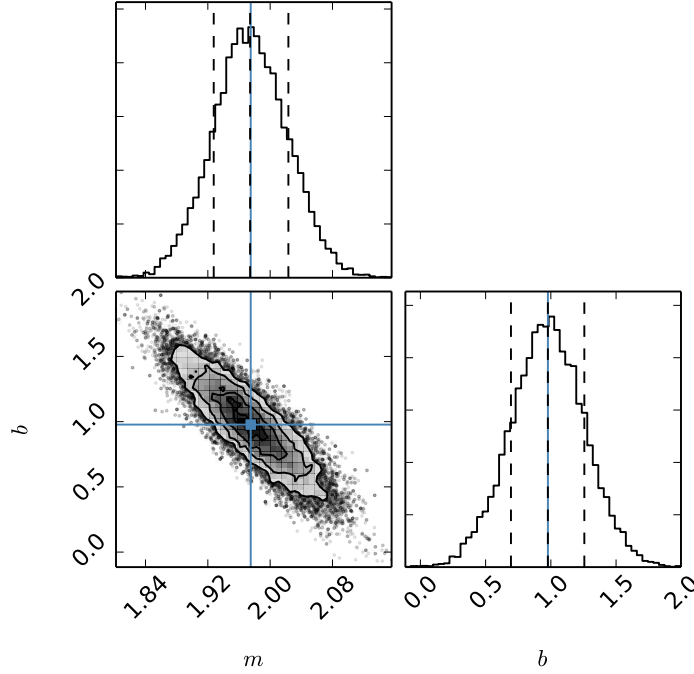


Figure 3.3: A triangle plot, showing the one-dimensional and two-dimensional posterior distributions for the MCMC run. The best-fitting parameters are indicated by the solid blue lines, while the median values and 1σ errors are given by the dashed black lines.

The scatter plot is a two-dimensional PD that demonstrates the level of correlation, or covariance, between pairs of parameters. Uncorrelated parameters will show circular scatter, or if the scaling of one parameter is dramatically different than the other, elongation in either the horizontal or vertical directions. Degenerate parameters manifest angled trends, reflecting that a change in one parameter can be well-compensated by altering the value of the other. Such is the case for the parameters m and b in our linear fitting example. The best-fit line given by the MCMC run is plotted in red in Figure 3.1, and agrees well with the original linear function.

For more complex models, such as that of our phase curve, the PD can become more complicated. In addition to heavily degenerate parameters, the PD

itself may take on a multi-modal appearance. Such a form indicates the presence of local maxima in the distribution. AI MCMC is not well suited to dealing with multi-modal PDs because of the aforementioned collectivism of the walkers. If a large portion of the walkers become lodged in a sharp local maximum, the remaining walkers will have a tendency to move towards this false best fit. Alternatively, if the walkers become divided between two maxima, their movement will be confined to the line collecting the two. The probability gradient connecting the two regions may be prohibitively low, preventing the collection from coalescing to the better fit. We seek an MCMC that can cope with such multi-modality, and thus ultimately decided against using a pure AI implementation.

3.1.3 Parallel-tempering algorithm

Parallel-tempering (PT) MCMC (Earl & Deem 2005; Foreman-Mackey et al. 2013) implements a modified version of the AI MCMC to remedy the latter’s difficulty dealing with multi-modal PDs by simultaneously running several MCMCs at different “temperatures.” Increasing the temperature flattens the probability distribution, allowing the walkers more freedom of movement in parameter space. The PT routine takes advantage of these multiple temperatures by periodically swapping walkers between different temperature runs, enabling the free-wielding nature of the high temperatures to trickle-down to the more judicious low temperatures. This mixing between broad and precise sampling seeks to minimize the complicating effect of multi-modality. We therefore implement a parallel-tempered MCMC from the `emcee` Python library (Foreman-Mackey et al. 2013) in our fitting formalism, as described in the following section.

3.2 Formalism

Our fitting process, or formalism, for a given light curve consists of a two-phase approach. The first phase provides a broad scan of the parameter space, and investigates different combinations of phase effects. The second phase acts as a followup analysis, meant to hone and refine the results of the first. We discuss both phases in greater detail below.

3.2.1 Phase one: model comparison

The light curves of star-planet systems all feature contributions from each of the four effects we model; however, the amplitudes of each effect will vary. When the amplitudes for one or more effects drops below the sensitivity of Kepler, the validity of a fitted model including those effects is questionable. To allow for such cases, we implement model testing between different combinations of phase effects. Including the null model, which only fits the variable flux offset term, there are sixteen distinct models, each with their own set of fitted parameters. For instance, for models that fit reflection, but not thermal emission, the R_p and A_g terms must be fit jointly, since no other equation constrains either parameter. The different models and their parameter sets are listed in Table 3.1.

Phase one of our fitting formalism consists of running two PT MCMCs in series for each model. For the first of the two runs, we initialize the walkers in a uniform distribution within the allowed bounds of parameter space. In the second run, we start the walkers in a Gaussian ball around the best-fitting parameter values from the first run, to enable a better-defined PD. For both sets of runs, we use five temperatures, 100 walkers, and 1,000 steps.

Model	Fitted parameters					
Re+Th+El+Do	M_p	—	R_p	A_g	T_{day}	
Th+El+Do	M_p	—	R_p	—	T_{day}	
Re+El+Do	M_p	$\sqrt{A_g} R_p$	—	—	—	
Re+Th+Do	M_p	—	R_p	A_g	T_{day}	
Re+Th+El	M_p	—	R_p	A_g	T_{day}	
El+Do	M_p	—	—	—	—	
Th+Do	M_p	—	R_p	—	T_{day}	
Th+El	M_p	—	R_p	—	T_{day}	
Re+Do	M_p	$\sqrt{A_g} R_p$	—	—	—	
Re+El	M_p	$\sqrt{A_g} R_p$	—	—	—	
Re+Th	—	—	R_p	A_g	T_{day}	
Do	M_p	—	—	—	—	
El	M_p	—	—	—	—	
Th	—	—	R_p	—	T_{day}	
Re	—	$\sqrt{A_g} R_p$	—	—	—	
null	—	—	—	—	—	

Table 3.1: Parameters that vary between models of different combinations of effects. Note that at least three effects, including both reflection and thermal, must be fit in order to fully characterize the planet (bolded model names). e , i , ω , M_0 , and f_0 are fit by all models.

From the second MCMC run for each of the sixteen models, we extract the minimum reduced chi-square value (χ^2_ν), and use these to determine which combination of effects describes the data best. Determining which model provides a statistically better fit requires considering both the quality of the fit and the complexity of the model. For two models of equal complexity (i.e., the same number of fitted parameters), the better fitting model is simply the one with the lesser χ^2_ν . If two models have different numbers of parameters, the F-test (cf. Bevington & Robinson 1992) provides a means to determine whether an increase in fit quality is justified by an increase in model complexity. In this manner, we rank the success of each model’s ability to fit the data.

3.2.2 Phase two: honing the fits

Phase two refines the fits of the top three, best-fitting models as given by phase one. For each model, we run an additional ten PT MCMCs with five temperatures, 100 walkers, and 1,000 steps. The initial walker positions for these ten runs are Gaussians centered at the set of best-fitting parameter values from the second run of phase one. These sets of ten runs allow an evaluation of the robustness of our overall fitting process: if all give relatively similar results, we can be confident the results represent the best-fitting model and parameter values.

3.3 Synthetic planet tests

To test and demonstrate our fitting formalism, we implement it on two synthetic light curves that contain a planetary phase-curve signal. The “true” parameters used to create these light curves are given in Table 3.2, and the light curves themselves in Figure 3.4. Note that both planets are hot Jupiters, orbiting the same type of star, at an inclination that would cause them to transit. For the purpose of this demonstration, we do not attempt to model and then excise transits and occultations. Instead, we construct the light curve solely from phase curve variations. To estimate Kepler-level noise, we add Gaussian noise characteristic of a similar Kepler target, realizing this is a rough approximation.

For purposes of fitting, we assume the period of the model planet is known, as is the case for confirmed planets. Using Newton’s version of Kepler’s third law, we derive the semi-major axis of the orbit from the period. We also take the stellar parameters in Table 3.2 to be known. We are thus left with, at most, nine free parameters to fit, in the case of the most complex model (Re+Th+El+Do). We apply our fitting formalism (discussed above) to both model planets’ data,

Model parameters	Planet 1	Planet 2
P [days]	1.75	5
e	0.05	0.05
i [deg]	85	85
ω [rad]	3.5	0
M_0 [rad]	1.2	1.2
M_p [M_J]	8	8
R_p [R_J]	2	2
A_g	0.15	0.1
$\sqrt{A_g} R_p$ [R_J]	0.77	0.63
T_{day} [K]	3500	2500
f_0	2×10^{-4}	2×10^{-4}
$M_\star$ [$M_\odot$]	2.05	2.05
$R_\star$ [$R_\odot$]	2.55	2.55
T_{eff} [K]	8500	8500

Table 3.2: Input parameters used to create synthetic light curves.

enforcing parameter bounds given in Table 3.3. The upper limit on the eccentricity is constrained by the limitation of our iterative series solution method for calculating the eccentric anomaly (see Chapter 2). Otherwise, the bounds are very general, and assume no prior knowledge of the planets. Further refinement of these bounds is possible. For instance, it is unlikely for a planet to have an eccentricity in excess of 0.1, and even more unlikely in the case of close-in planets, which tend to produce stronger phase effects. We will return to refining bounds based on a priori knowledge in Chapter 4. For now, however, we choose to leave them relatively unconstrained for these synthetic tests, if for no other reason than to probe the effectiveness of our fitting formalism over a larger parameter space.

Model parameters	Bounds
e	$[1 \times 10^{-3}, 0.6]$
i [deg]	$[0, 90]$
ω [rad]	$[0, 2\pi]$
M_0 [rad]	$[0, 2\pi]$
M_p [M _J]	$[0.1, 20]$
R_p [R _J]	$[0.1, 10]$
A_g	$[0, 1]$
$\sqrt{A_g} R_p$ [R _J]	$[0, 10]$
T_{day} [K]	$[0, 6000]$
f_0	$[-1 \times 10^{-2}, 1 \times 10^{-2}]$

Table 3.3: Parameter bounds used for fitting the synthetic light curves. Note that while the i bounds are given in degrees, for fitting purposes, we sample $\cos i$ on $[0, 1]$ to ensure uniform spherical sampling.

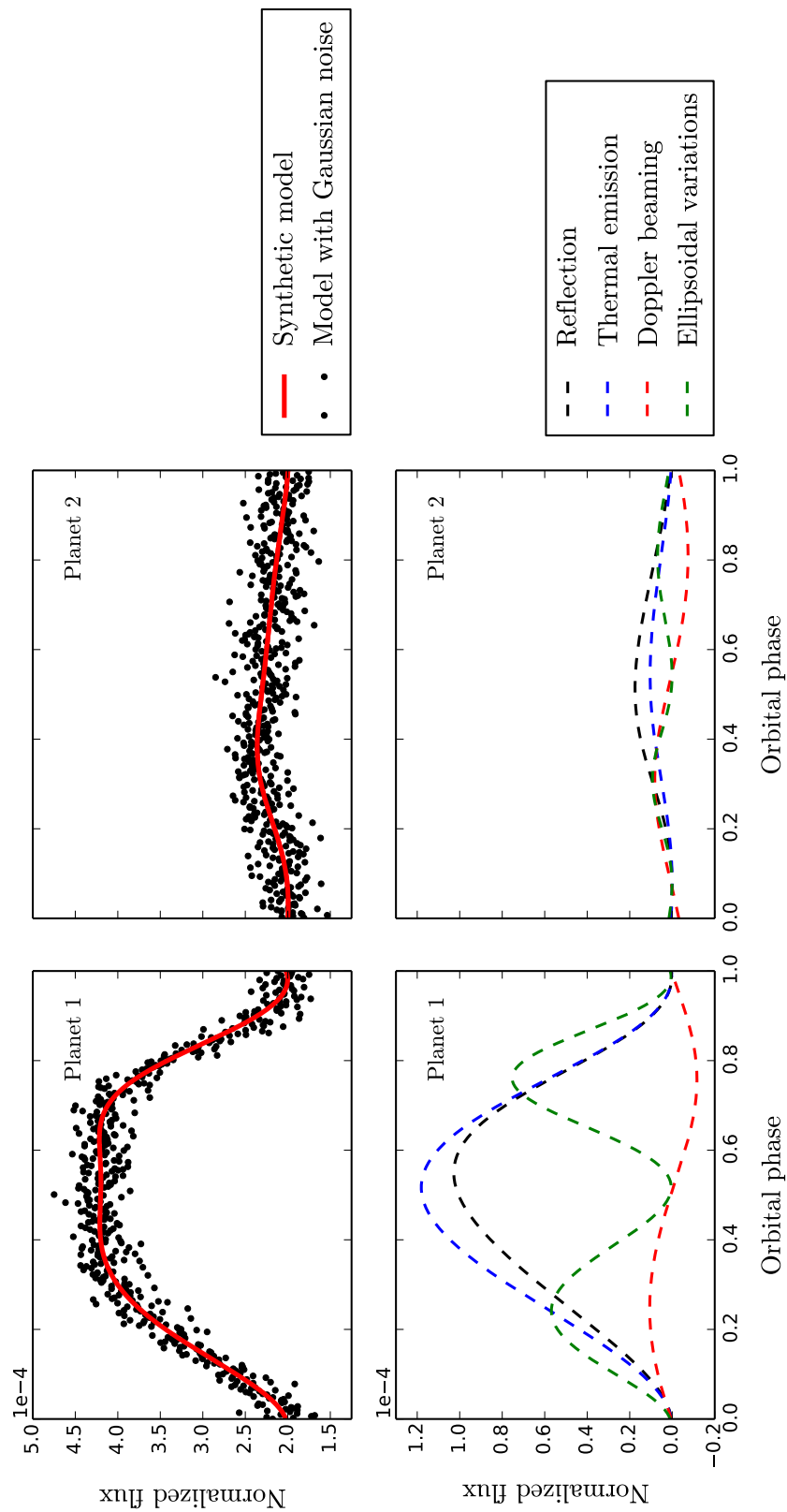


Figure 3.4: Top panel: Synthetic models with and without added noise. Bottom panel: Constituent phase effects.

3.3.1 Planet one

Table 3.4 gives the results of model comparison for planet one. Statistically, the best-fitting model is that which only consists of a reflected component. The four-effect model with which we created the synthetic data does not rank in the top three statistically significant models, suggesting model degeneracy. We therefore include the four-component Re+Th+El+Do model in the phase two runs for diagnostic purposes, in addition to the three most significant models.

Model	χ^2_ν	Model	χ^2_ν
Th+El+Do	1.017	Re	1.111
Re+Th+El+Do	1.019	Re+El+Do	1.040
Th+El	1.027	Re+El	1.061
Re+Th+El	1.029	Re+Do	1.109
Re+El+Do	1.040	Th+El+Do	1.017
Re+El	1.061	Th+El	1.027
Re+Do	1.109	Th	1.191
Re	1.111	Re+Th+El+Do	1.019
Re+Th+Do	1.115	Re+Th	1.118
Re+Th	1.118	Th+Do	1.194
Th	1.191	Re+Th+El	1.029
Th+Do	1.194	Re+Th+Do	1.115
El+Do	3.623	El+Do	3.623
El	3.850	El	3.850
Do	12.052	Do	12.052
null	19.929	null	19.929

(a)
(b)

Table 3.4: Model testing results for planet one. (a) Models sorted by reduced chi-square values. (b) Models and corresponding reduced chi-square values sorted by statistical significance, as given by the F-test.

The phase two results for planet one are given in Table 3.5, for both the Re and Re+Th+El+Do models. A cursory comparison with the true parameter values (Table 3.2) immediately reveals poor agreement for the Re model results. Only

the fitted i and $\sqrt{A_g}R_p$ median values agree to within 3σ with the true values. The four-effect model returned remarkably better parameters, all of which agree within 1σ of the true parameters except for e , M_p , A_g , and T_{day} , which agree to within 2σ .

The triangle plots of the best runs for the Re and four-effect models (Figures 3.5 and 3.6, respectively) give insight into parameter degeneracies. For the four-component model, PD scatter plots indicate degeneracy between R_p and A_g , and R_p and T_{day} , which manifest in both the best fit values and median values as depressed radius and albedo values, and increased dayside temperatures. Such a result is understandable considering that these three parameters have similar influences on the reflection and thermal component amplitudes, which at such small eccentricities are nearly indistinguishable (see Chapter 2).

The plots of the best fitting models (Figures 3.7 and 3.8) help justify the statistical significance of the single-component Re model over the four-component model. The synthetic data's unimodal appearance can be well-reproduced by a unimodal reflection curve, albeit for parameter values that differ significantly from the true values. Since the χ^2_ν values are so similar, the simpler model is preferred.

Parameter	Best	Median
<i>Re</i>		
e	0.1108	$0.1259^{+0.0086}_{-0.0085}$
i [deg]	23.4	$36.6^{+19.1}_{-8.0}$
ω [rad]	4.635	$4.591^{+0.026}_{-0.034}$
M_0 [rad]	0.199	$0.234^{+0.030}_{-0.029}$
$\sqrt{A_g} R_p$ [R _J]	2.66	$1.90^{+0.36}_{-0.41}$
<i>Re+Th+El+Do</i>		
e	0.0720	$0.0694^{+0.0083}_{-0.0099}$
i [deg]	76	74^{+11}_{-12}
ω [rad]	3.36	$3.41^{+0.15}_{-0.15}$
M_0 [rad]	1.34	$1.31^{+0.14}_{-0.15}$
M_p [M _J]	8.77	$8.90^{+1.57}_{-0.79}$
R_p [R _J]	2.38	$1.95^{+0.50}_{-0.34}$
A_g	0.026	$0.050^{+0.063}_{-0.035}$
T_{day} [K]	3610	3810^{+320}_{-280}

Table 3.5: Results of the phase two runs for planet one for the most statistically significant model, as well as the four-effect model. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit.

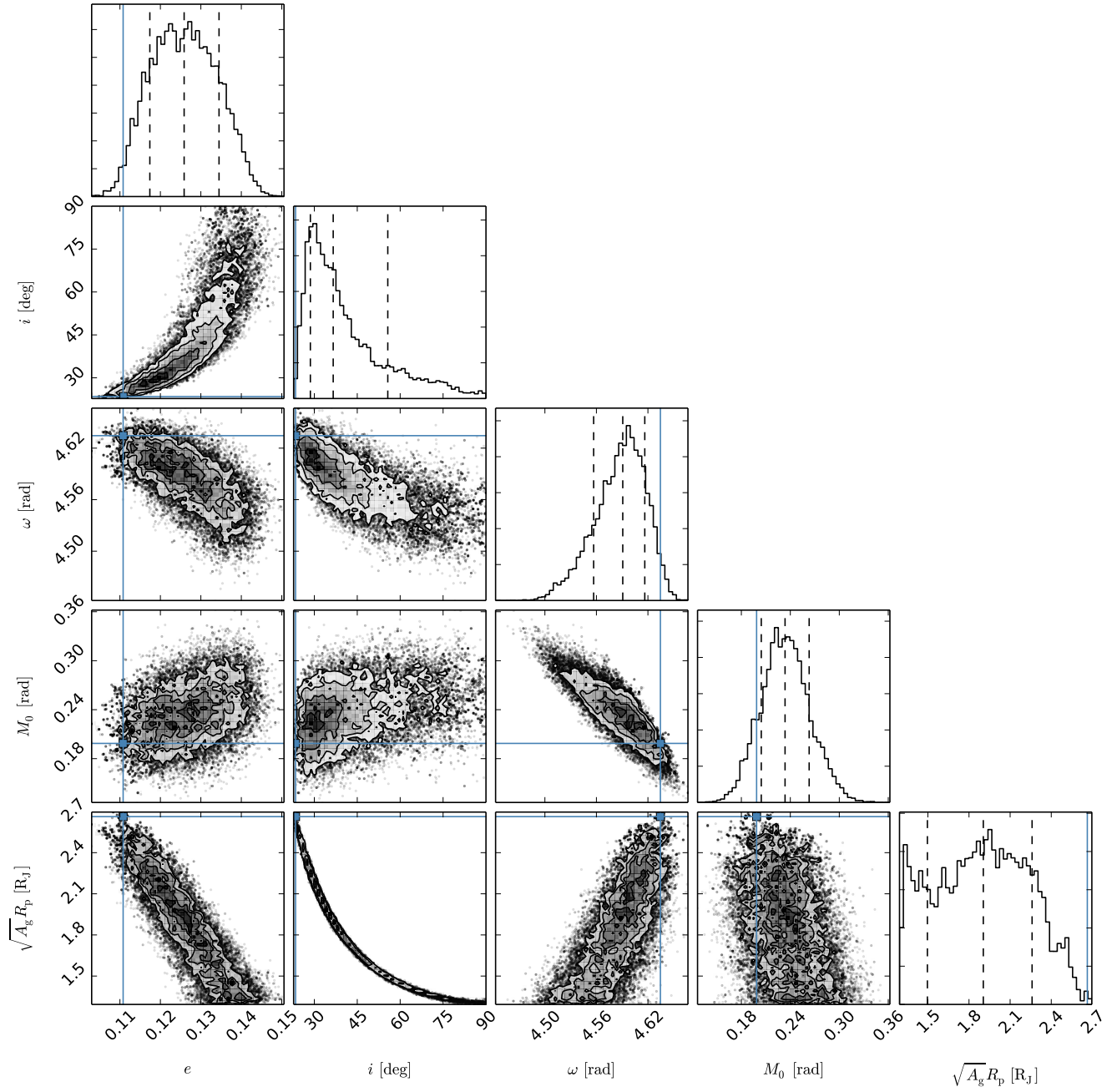


Figure 3.5: Triangle plot of the phase two run that yielded the best fit of the Re model for planet one.

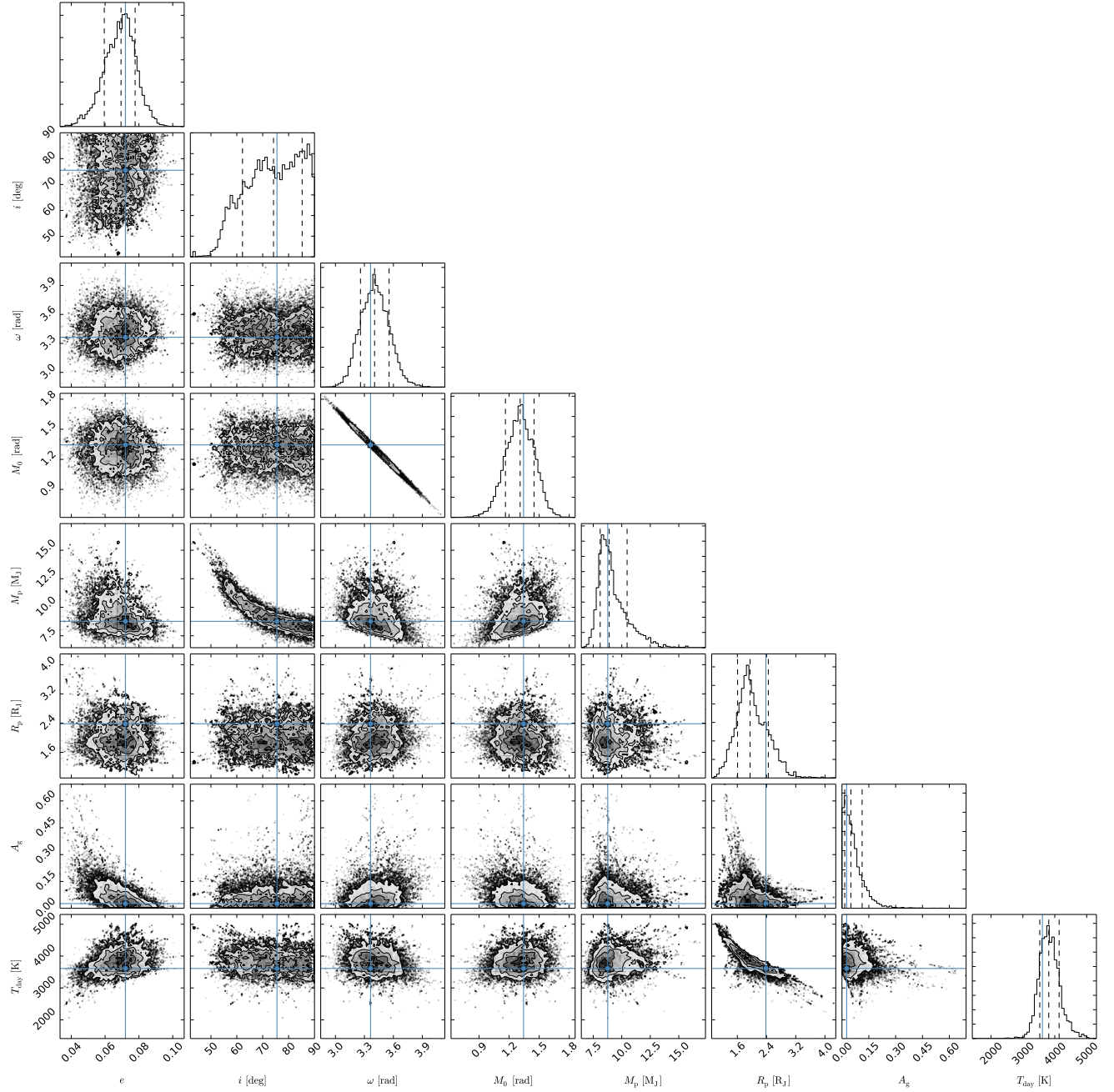


Figure 3.6: Triangle plot of the phase two run that yielded the best fit of the Re+Th+El+Do model for planet one.

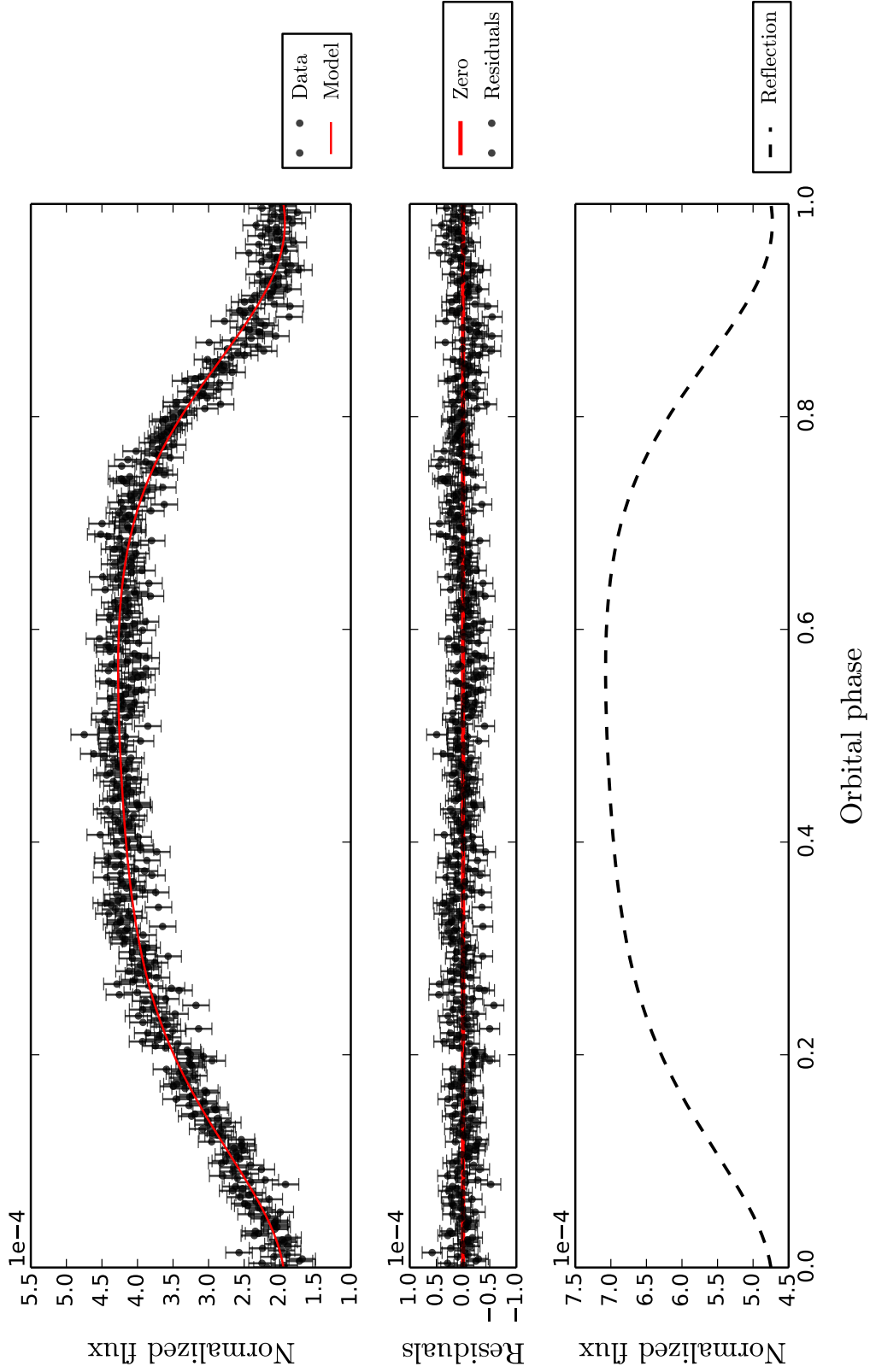


Figure 3.7: Top panel: Best-fitting Re model overlaid on planet one data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

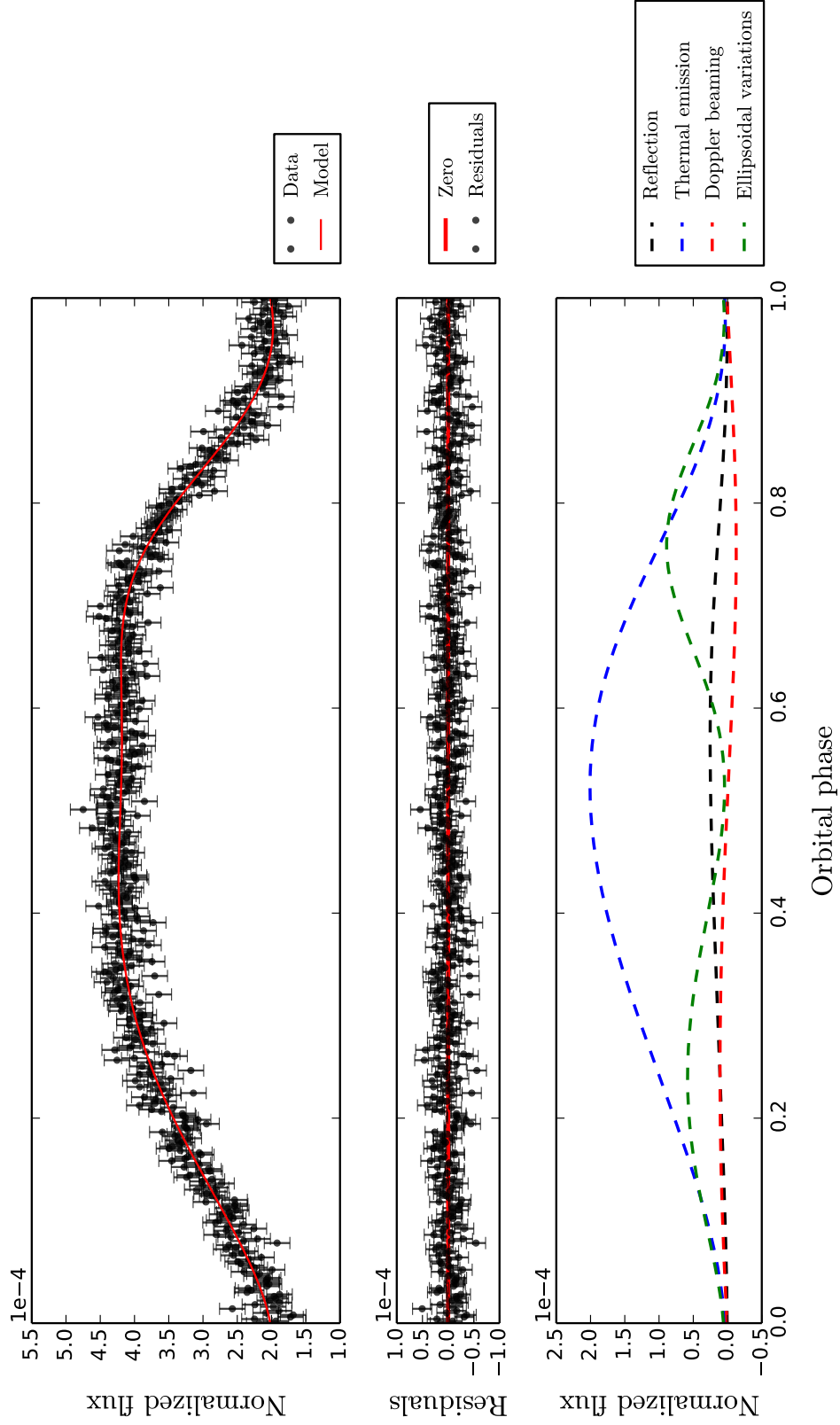


Figure 3.8: Top panel: Best-fitting Re+Th+El+Do model overlaid on planet one data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

3.3.2 Planet two

Phase one of the formalism for planet two gives the Doppler beaming-only model as the best-fitting, and the most statistically-significant model (see Table 3.6). As in the case of planet one, the Re+Th+El+Do model does not make the top three most statistically significant models, but we choose to include it in phase two nonetheless. The phase two results are similar to those of planet one in that the median parameters for the four-effect model are much closer to the true parameter values. In the Do model, for instance, the fitted median value for M_p is more than double the true value, as the mass of the planet must increase to replicate the curve created from four effects with just one (see Figures 3.9 and 3.10).

From both the 1σ errors on the median values and the corresponding triangle plot (Figure 3.12), however, it becomes clear that the four-component model parameters for planet two are less constrained than those for planet one. This decrease in constraint from planet one to planet two likely results from the decrease in signal strength between the two planets. Indeed, planet one orbits its star with a period less than half that of planet two, and thus has relatively amplified phase effects. The fit results for planet two also reveal degeneracies between the R_p , A_g , and T_{day} . For both the median and best-fit values, the undervaluation of the planetary radius is compensated by an inflation in the values for albedo and temperature.

Model	χ^2_ν	Model	χ^2_ν
Do	1.0116	Do	1.0116
Re	1.0117	Re	1.0117
Re+El	1.0121	El+Do	1.0194
Re+El+Do	1.0136	El	1.0274
Th	1.0136	Re+El	1.0121
Re+Do	1.0138	Re+El+Do	1.0136
Th+El+Do	1.0147	Th	1.0136
Th+El	1.0154	Re+Do	1.0138
Th+Do	1.0158	Th+El+Do	1.0147
Re+Th	1.0158	Th+El	1.0154
Re+Th+El	1.0164	Th+Do	1.0158
Re+Th+El+Do	1.0168	Re+Th	1.0158
Re+Th+Do	1.0177	Re+Th+El	1.0164
El+Do	1.0194	Re+Th+El+Do	1.0168
El	1.0274	Re+Th+Do	1.0177
null	1.3837	null	1.3837

(a)
(b)

Table 3.6: Same as Table 3.4, but for planet two.

Parameter	Best	Median
<i>Do</i>		
e	0.349	$0.340^{+0.067}_{-0.069}$
i [deg]	84	72^{+12}_{-13}
ω [rad]	4.89	$4.87^{+0.23}_{-0.23}$
M_0 [rad]	1.72	$1.74^{+0.19}_{-0.18}$
M_p [M _J]	16.5	$17.2^{+1.6}_{-1.5}$
<i>Re+Th+El+Do</i>		
e	0.141	$0.105^{+0.082}_{-0.064}$
i [deg]	80	64^{+19}_{-19}
ω [rad]	6.27	$5.82^{+0.33}_{-0.60}$
M_0 [rad]	1.27	$1.67^{+0.42}_{-0.31}$
M_p [M _J]	7.6	$9.6^{+3.8}_{-4.6}$
R_p [R _J]	0.45	$0.66^{+0.43}_{-0.30}$
A_g	0.24	$0.50^{+0.33}_{-0.32}$
T_{day} [K]	4400	3800^{+1200}_{-1500}

Table 3.7: Same as Table 3.5, but for planet two.

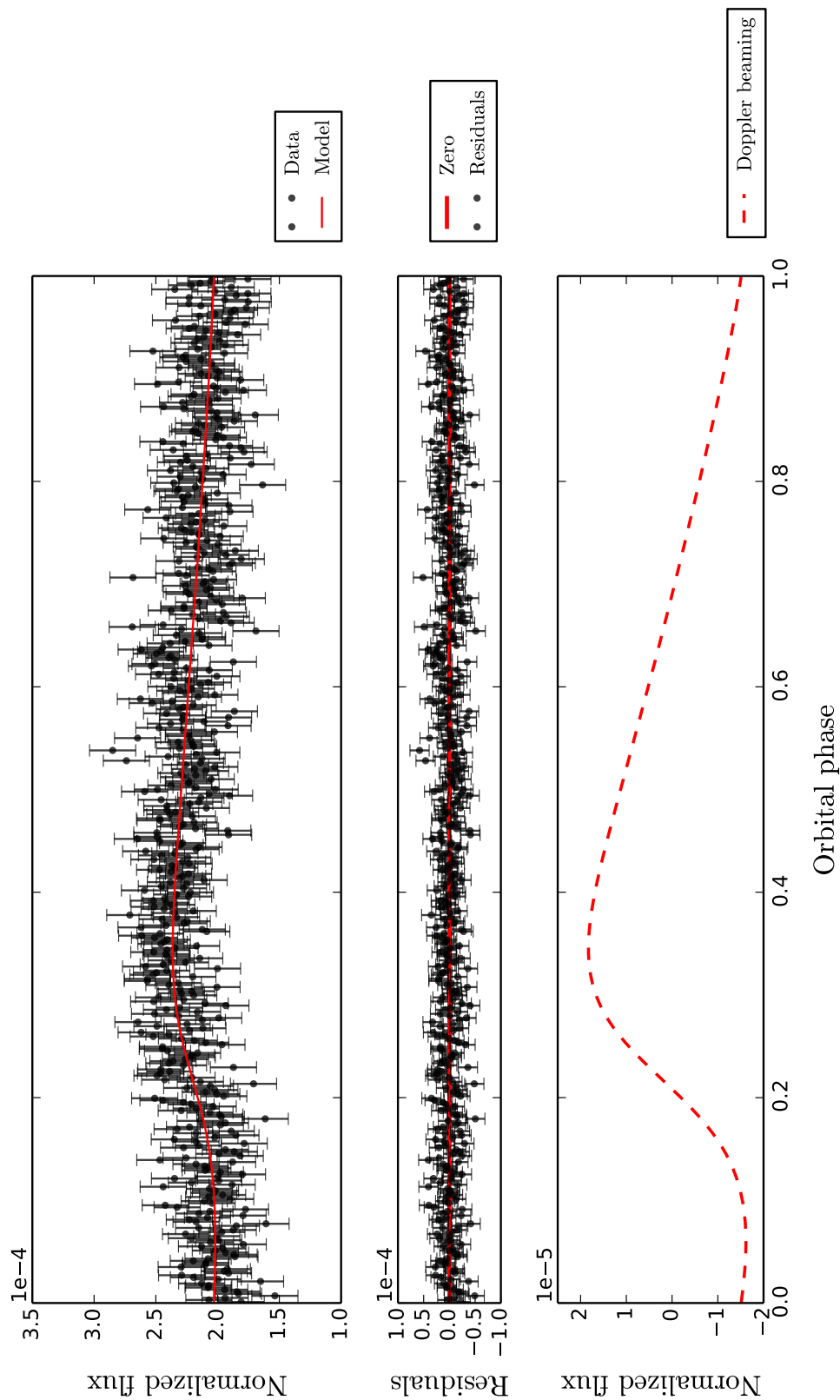


Figure 3.9: Top panel: Best-fitting Do model overlaid on planet two data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

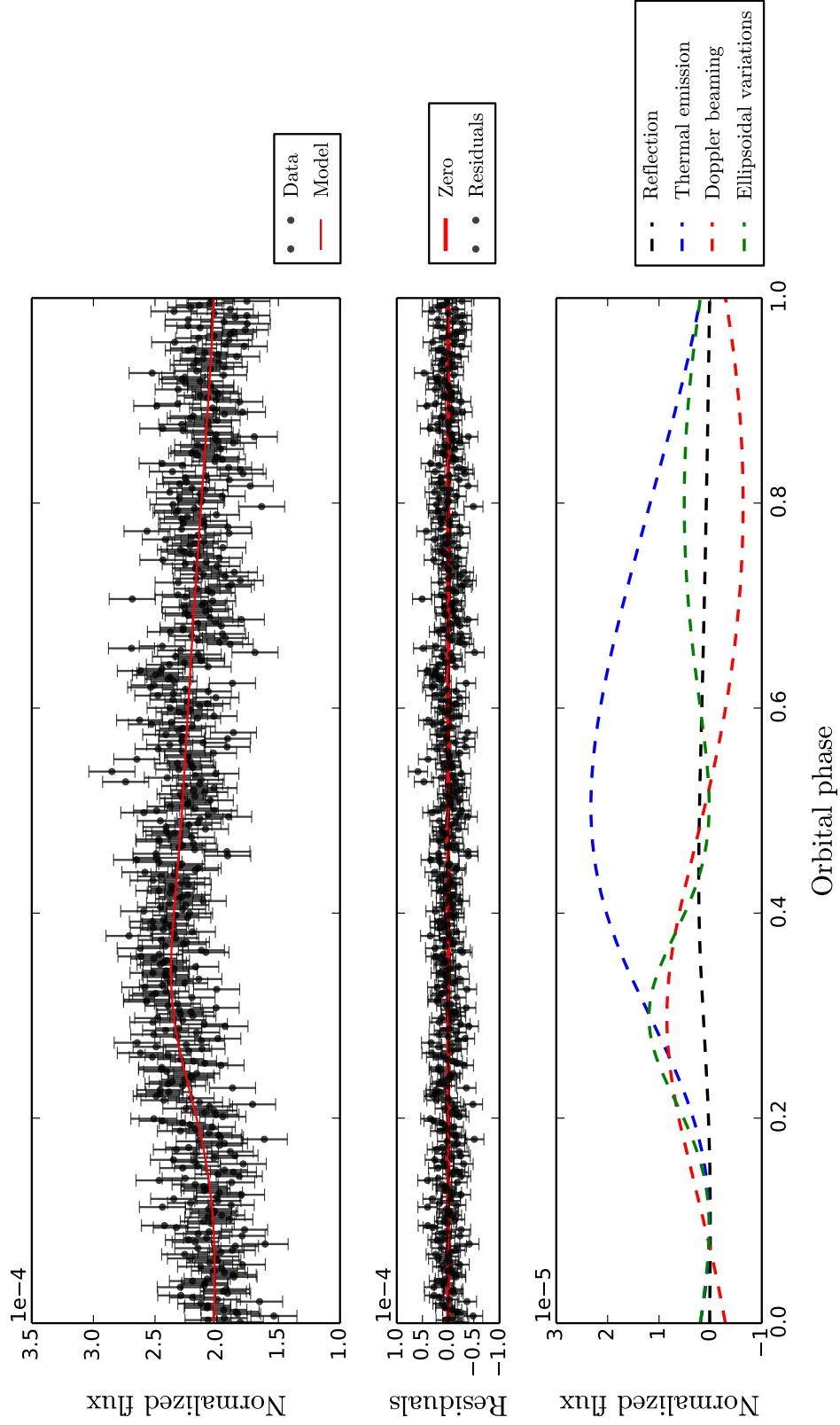


Figure 3.10: Top panel: Best-fitting Re+Th+El+Do model overlaid on planet two data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

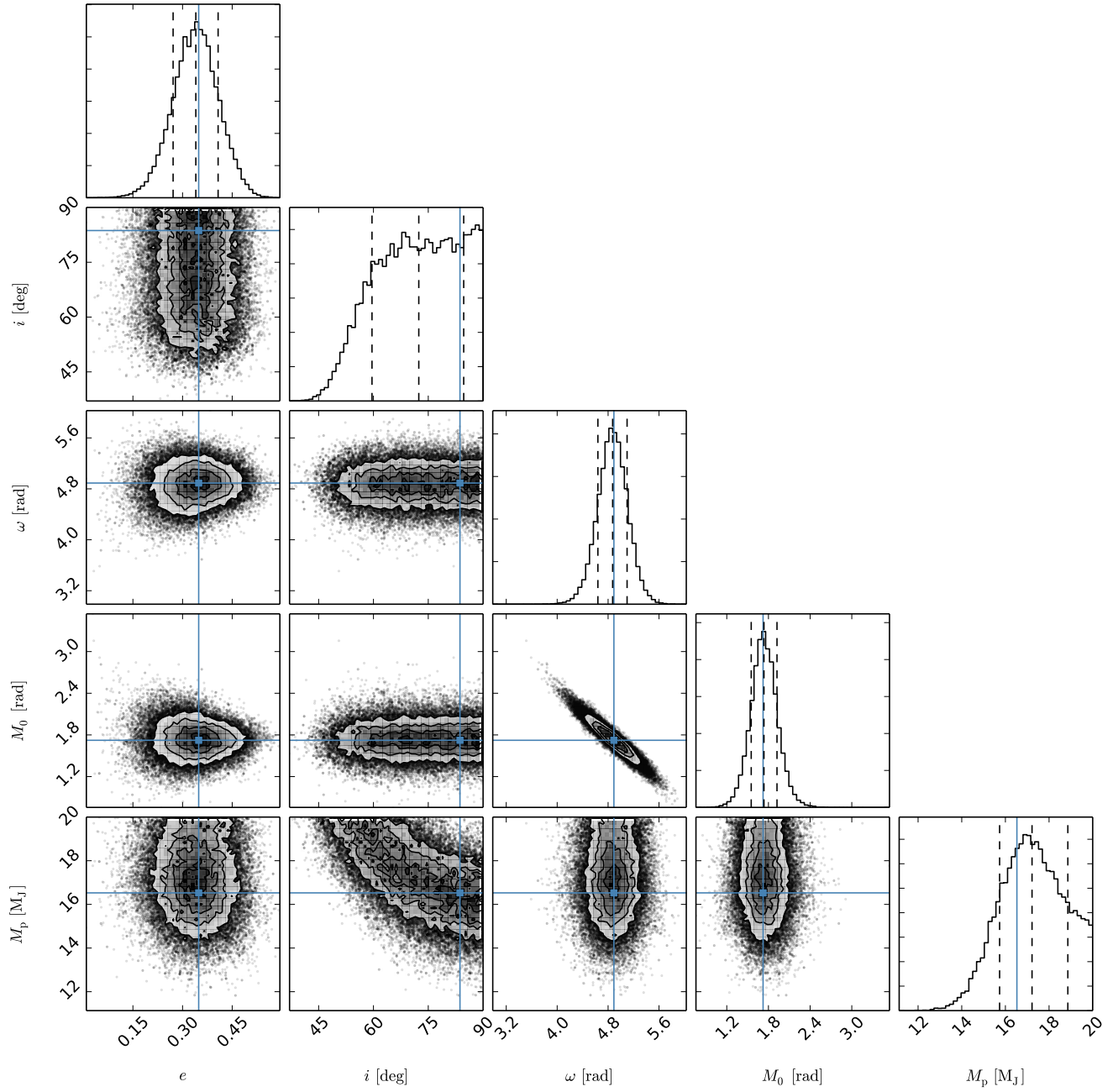


Figure 3.11: Triangle plot of the phase two run that yielded the best fit of the Do model for planet two.

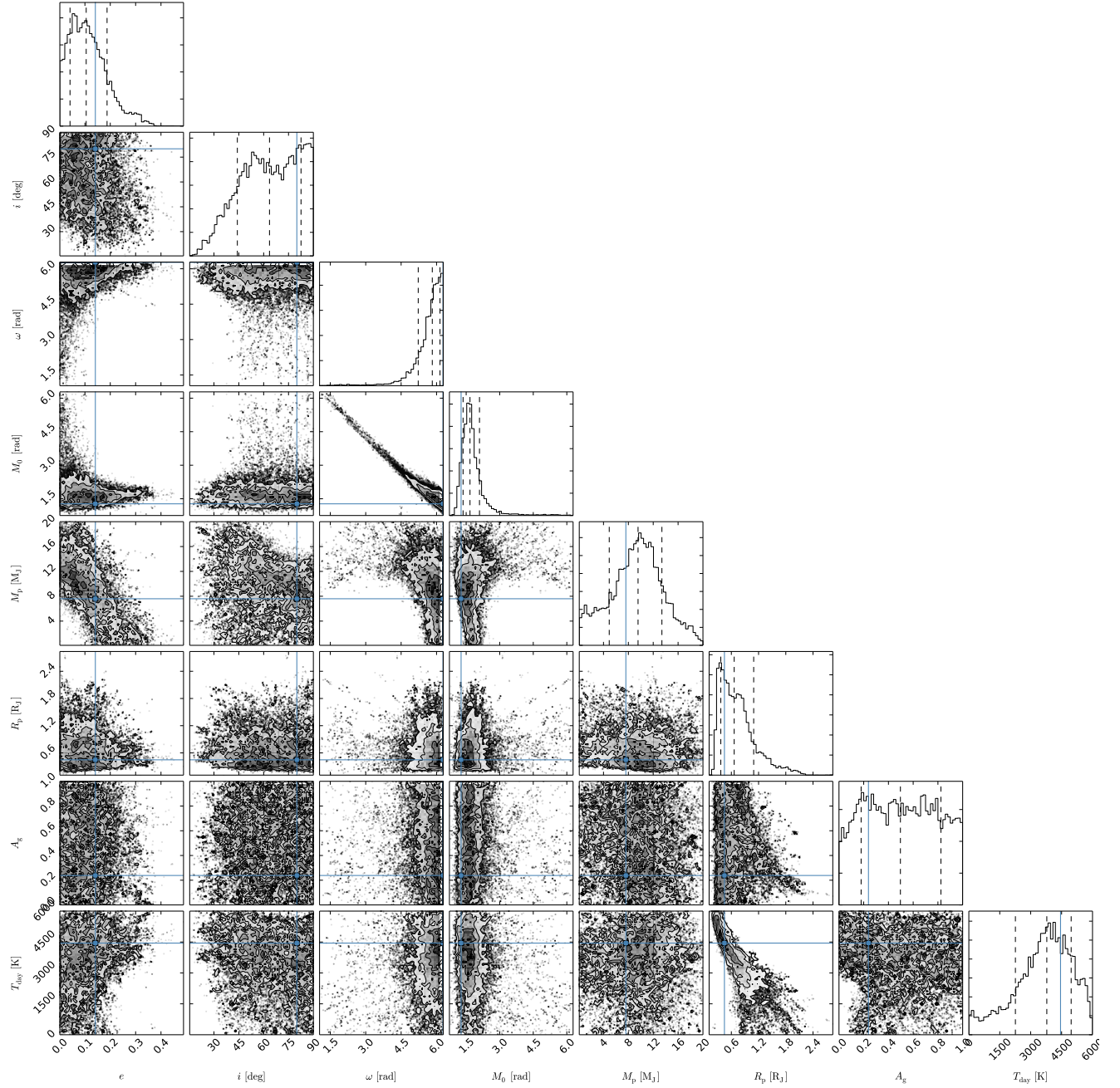


Figure 3.12: Triangle plot of the phase two run that yielded the best fit of the Re+Th+El+Do model for planet two.

3.3.3 Conclusions

Fitting the synthetic light curves raises several issues pertinent to fitting real data. Chief among these is degeneracy between different phase curve models. It is clear from our tests that it is possible—if not common—for models consisting of only one effect to be more statistically significant than those fitting multiple effects, even when we know those additional effects are present. Having some prior knowledge of the planet can assist in evaluating whether a given model makes sense. For instance, if we are analyzing a known transiting/occulting planet, we will have a general idea of what the eccentricity of its orbit is from the separation between transit and occultation. If we allow the eccentricity to vary freely over generous bounds in the fitting, and a model gives an e that deviates significantly from the eccentricity we expect, we can question the validity of that model. Alternatively, we can restrict the bounds of the eccentricity, to prevent degeneracy with models that give physically unrealistic fits.

The synthetic data tests also demonstrate a parameter degeneracy between R_p , A_g , and T_{day} . As these parameters have broadly the same constructive effect on the reflection and thermal components, we are not surprised by the covariance in the fit results. As with the model degeneracy, however, a priori knowledge can help break parameter degeneracy. Again, for a known transiting exoplanet, we can determine the planetary radius, and thereby narrow the bounds of that parameter. While this may not completely resolve the degeneracy between R_p , A_g , and T_{day} , it will prevent fits from giving a planetary radius that is known to be untrue.

We also must recognize that when the phase curve signal strength decreases to a level comparable to the noise in the data, the fit results will lose their

precision. This selection effect, which makes phase curve analysis most effective for more massive planets closer to their stars (like planet one), is common to both the transit and radial velocity techniques. Our analysis of planet two suffers from this bias, despite the fact that it orbits its star at a rather close, five-day orbital period and has a substantial mass. Detecting hot Jupiters further out, or even super-Earths and Neptunes with close orbits will be challenging.

Chapter 4

Kepler K1 Planets

While testing our fitting formalism on synthetic data proved illuminating, the process is necessarily self-consistent. We would like to compare results of our fitting process on actual Kepler data to other published analyses, particularly those involving phase curves. To this end, we selected several K1 planets whose transits and phase curves have been analyzed, and ran them through our fitting formalism. This chapter presents our data reduction technique for processing Kepler K1 data, followed by the results of fitting for each of these planets.

4.1 Data reduction

In an attempt to correct for systematics, the Kepler team developed a pipeline that uses engineering data to detrend the raw Kepler light curves (Thompson & Fraquelli 2014). The resulting data, denoted presearch data conditioning (PDC) light curves, generally show much less scatter than the uncorrected data. Nonetheless, we opt to implement further data conditioning to minimize the spread in data, especially in light of the model degeneracy issues raised in our synthetic data tests (see Chapter 3).

We first remove any points flagged by the Kepler data pipeline as potentially having sub-optimal quality (e.g., lack of fine pointing, cosmic ray event, etc.), as well as any remaining non-finite flux values. As the targets we will fit have

confirmed transiting planets, and thus published values for e , i , and R_p from transit analyses, we calculate the durations and separation of the transit and occultation, and excise these regions of the light curve. Since transit analysis does not provide a well constrained e , we extend the calculated durations by a fractional length to ensure the entire transit/occultation is fully removed.

To further remove any remaining long-term systematic trends, we fit a cubic spline to each quarter of data, with knots every two orbital periods to minimize astrophysical signal removal. We divide out the ratio of the spline over the median of the spline, so as to preserve the same flux level. Removing short-term trends that occur within a given orbit without simultaneously removing the phase curve signal is incredibly difficult, so we opt to excise egregious orbits by sigma clipping. We split each orbit into halves, and keep only those orbits for which the medians of both halves and the median of the whole orbit lie within 2σ of the median for that quarter.

We also sigma-clip on a point-by-point basis, removing data that lies more than 3σ from its quarter’s median. If more than fifty percent of an orbit is removed by this round of sigma clipping, we consider it a “bad” orbit, and remove it entirely. We normalize each quarter by subtracting and then dividing by its median. Finally, we bin the combined data from all quarters to 1/400th the published orbital period, and phase fold over the published transit ephemeris such that orbital phase $\phi = 0$ corresponds to mid-transit.

4.2 Kepler-13b

Kepler-13b has one of the strongest photometric signals of the confirmed Kepler exoplanets, so much so that Shporer et al. (2011) have shown it is detectable solely

from its phase curve variations. Several analyses (e.g., Placek et al. 2014, Esteves et al. 2013, Shporer et al. 2011) of its phase variations have been published, making Kepler-13b a natural choice with which to test the accuracy of our model and fitting formalism. Table 4.1 gives the published stellar parameters and orbital period (derived from transit analysis) we use in our data reduction and fitting. We also restrict the bounds of the inclination and planetary radius to 3σ about the published values from transit analyses (see Table 4.1), as preliminary tests consistently yielded inclinations that would prohibit a transit, as well as unrealistic radius values. Bounds used for the remaining parameters are given in Table 4.2.

Parameter	Published value	Reference
$M_{\star} [M_{\odot}]$	2.05	Szabó et al. 2011
$R_{\star} [R_{\odot}]$	2.55	Szabó et al. 2011
$T_{\text{eff}} [\text{K}]$	8500	Szabó et al. 2011
$P [\text{days}]$	1.7635877	Batalha et al. 2013
$i [\text{deg}]$	81.8 ± 0.2	Masuda 2015
$R_p/R_{\star}$	0.07799 ± 0.00020	Batalha et al. 2013

Table 4.1: Published values used in our data reduction and fitting for Kepler-13b.

Table 4.3 gives the results of model testing for Kepler-13b. Unlike the synthesized planets, our fitting formalism returned more complicated combinations of three and four effects as the top three most statistically-significant models. By comparison, Placek et al. (2014) give Th+El+Do and Re+Th+El+Do as the favored models to describe Kepler-13b’s phase curve. The median values for the fitted parameters for each model (see Table 4.4) generally agree well with previous phase curve analyses, summarized in Table 4.5. For each of our three most statistically-significant models, all parameters except e and T_{day} agree to within $1 - 2\sigma$ of the corresponding values of each of the three published works. As each of the publications in Table 4.5 uses different combinations of effects—Th+El+Do,

Model parameters	Bounds
e	$[0, 0.6]$
ω [rad]	$[0, 2\pi]$
M_0 [rad]	$[0, 2\pi]$
M_p [M_J]	$[0.1, 20]$
A_g	$[0, 1]$
$\sqrt{A_g} R_p$ [R_J]	$[0, 10]$
T_{day} [K]	$[0, 6000]$
f_0	$[-1 \times 10^{-2}, 1 \times 10^{-2}]$

Table 4.2: Parameter bounds used for fitting the Kepler light curves. Note that while the i bounds are given in degrees, for fitting purposes, we sample $\cos i$ on $[0, 1]$ to ensure uniform spherical sampling.

Re+Th+El+Do, and Re+El+Do for Placek et al. (2014), Esteves et al. (2013), and Shporer et al. (2011), respectively—we conclude there is significant model degeneracy between these combinations.

Figures 4.1, 4.2, and 4.3, which plot the best-fitting models, further demonstrate this degeneracy. Throughout all three cases, the overall shape of the combined curve remains relatively constant, because swapping a unimodal reflection curve for a unimodal thermal curve of the same amplitude does not dramatically alter the composite light curve. Similarly, replacing either reflection or thermal with a combination of the two, each with a depressed amplitude, yields a similar shape. Indeed, our albedo and temperature values of the Re+Th+El+Do model are lower than those of our three-effect models.

As previously mentioned, some of our eccentricity values disagree with those of Placek et al. (2014). Our value from the comparable model (i.e., Th+El+Do) agrees to within 3σ , however, the values from our other two models fall below 3σ of Placek et al. (2014)’s values. As Szabó et al. (2011) argue for a circular orbit,

our lower e values compared to Placek et al. (2014) do not seem problematic. We also have discrepancies with T_{day} . While our values for temperature agree within $1 - 2\sigma$ with those of Placek et al. (2014), they do not agree to within 3σ of Esteves et al. (2013). A reason for this disagreement is not immediately clear, however, that Esteves et al. (2013) model the occultation may provide additional constraints on their temperature value. Our comparatively small error values for temperature may also play a role in this discrepancy.

Model	χ^2_ν	Model	χ^2_ν
Re+Th+El+Do	1.2640	Re+El+Do	1.4024
Re+El+Do	1.4024	Re+Th+El+Do	1.2640
Th+El+Do	1.5289	Th+El+Do	1.5289
Re+El	7.1069	Re+El	7.1069
Re+Th+El	7.1538	Th+El	7.2094
Th+El	7.2094	Re+Th+El	7.1538
Re+Do	17.7702	Re	18.3903
Re+Th+Do	17.8883	Re+Do	17.7702
Re	18.3903	Re+Th	18.5122
Re+Th	18.5122	Re+Th+Do	17.8883
Th	33.0279	Th	33.0279
Th+Do	33.1369	Th+Do	33.1369
El	48.7544	El	48.7544
El+Do	49.1299	El+Do	49.1299
Do	94.6318	Do	94.6318
null	302.8698	null	302.8698

(a)
(b)

Table 4.3: Model testing results for Kepler-13b. (a) Models sorted by reduced chi-square values. (b) Models and corresponding reduced chi-square values sorted by statistical significance, as given by the F-test.

Parameter	Best	Median
<i>Re+El+Do</i>		
e	0.03091	$0.03096^{+0.00061}_{-0.00058}$
i [deg]	82.24	$81.82^{+0.40}_{-0.41}$
ω [rad]	3.346	$3.349^{+0.051}_{-0.051}$
M_0 [rad]	1.322	$1.318^{+0.050}_{-0.051}$
M_p [M _J]	7.716	$7.721^{+0.050}_{-0.049}$
$\sqrt{A_g} R_p$ [R _J]	0.6806	$0.6810^{+0.0038}_{-0.0037}$
A_g	—	$0.1237^{+0.0015}_{-0.0015}$
<i>Re+Th+El+Do</i>		
e	0.0385	$0.0385^{+0.0013}_{-0.0012}$
i [deg]	81.62	$81.77^{+0.43}_{-0.39}$
ω [rad]	3.383	$3.380^{+0.041}_{-0.041}$
M_0 [rad]	1.285	$1.287^{+0.041}_{-0.040}$
M_p [M _J]	7.647	$7.644^{+0.047}_{-0.046}$
R_p [R _J]	1.9355	$1.9361^{+0.0099}_{-0.0101}$
A_g	0.0579	$0.0593^{+0.0070}_{-0.0075}$
T_{day} [K]	3045	3035^{+49}_{-48}
<i>Th+El+Do</i>		
e	0.04656	$0.04664^{+0.00093}_{-0.00093}$
i [deg]	81.21	$81.69^{+0.45}_{-0.35}$
ω [rad]	3.358	$3.367^{+0.034}_{-0.032}$
M_0 [rad]	1.307	$1.298^{+0.032}_{-0.034}$
M_p [M _J]	7.574	$7.550^{+0.044}_{-0.044}$
R_p [R _J]	1.945	$1.936^{+0.010}_{-0.010}$
T_{day} [K]	3331.2	$3335.9^{+6.4}_{-6.5}$

Table 4.4: Results of the phase two runs for Kepler-13b for the most statistically significant models. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit. Since the Re+El+Do model fits only the effective radius, A_g was calculated using the fitted $\sqrt{A_g} R_p$ and published R_p values.

Parameter	Placek et al. 2014	Esteves et al. 2013	Shporer et al. 2011
e	0.062 ± 0.005	—	—
ω [rad]	3.42 ± 0.10	—	—
M_0 [rad]	1.23 ± 0.10	—	—
M_p [M _J]	7.10 ± 0.60	7.95 ± 0.27	9.2 ± 1.1
A_g	—	$0.092^{+0.034}_{-0.041}$	—
T_{day} [K]	3492.4 ± 340.4	3558^{+53}_{-63}	—

Table 4.5: Published phase curve analysis values for Kepler-13b.

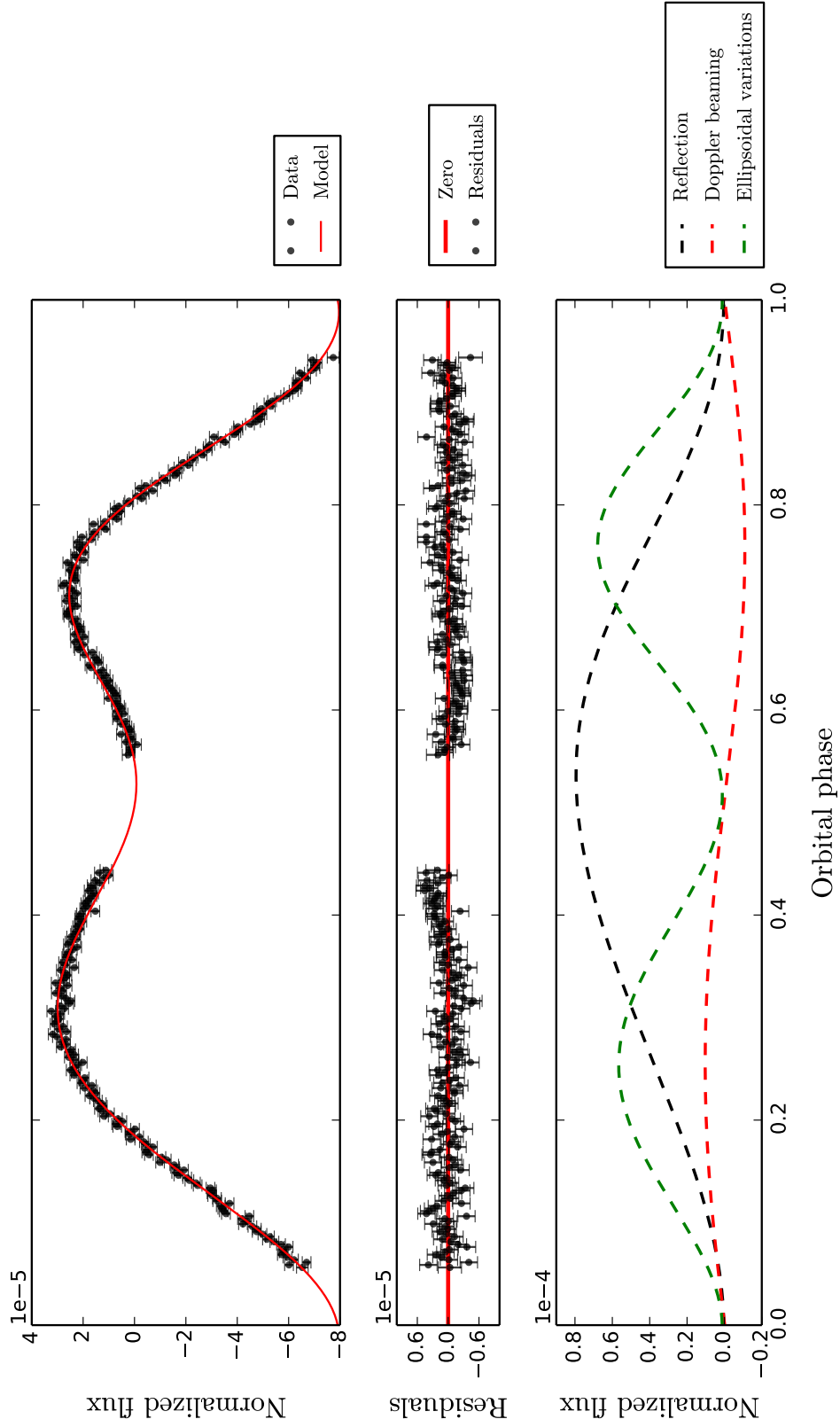


Figure 4.1: Top panel: Best-fitting Re+El+Do model overlaid on Kepler-13b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

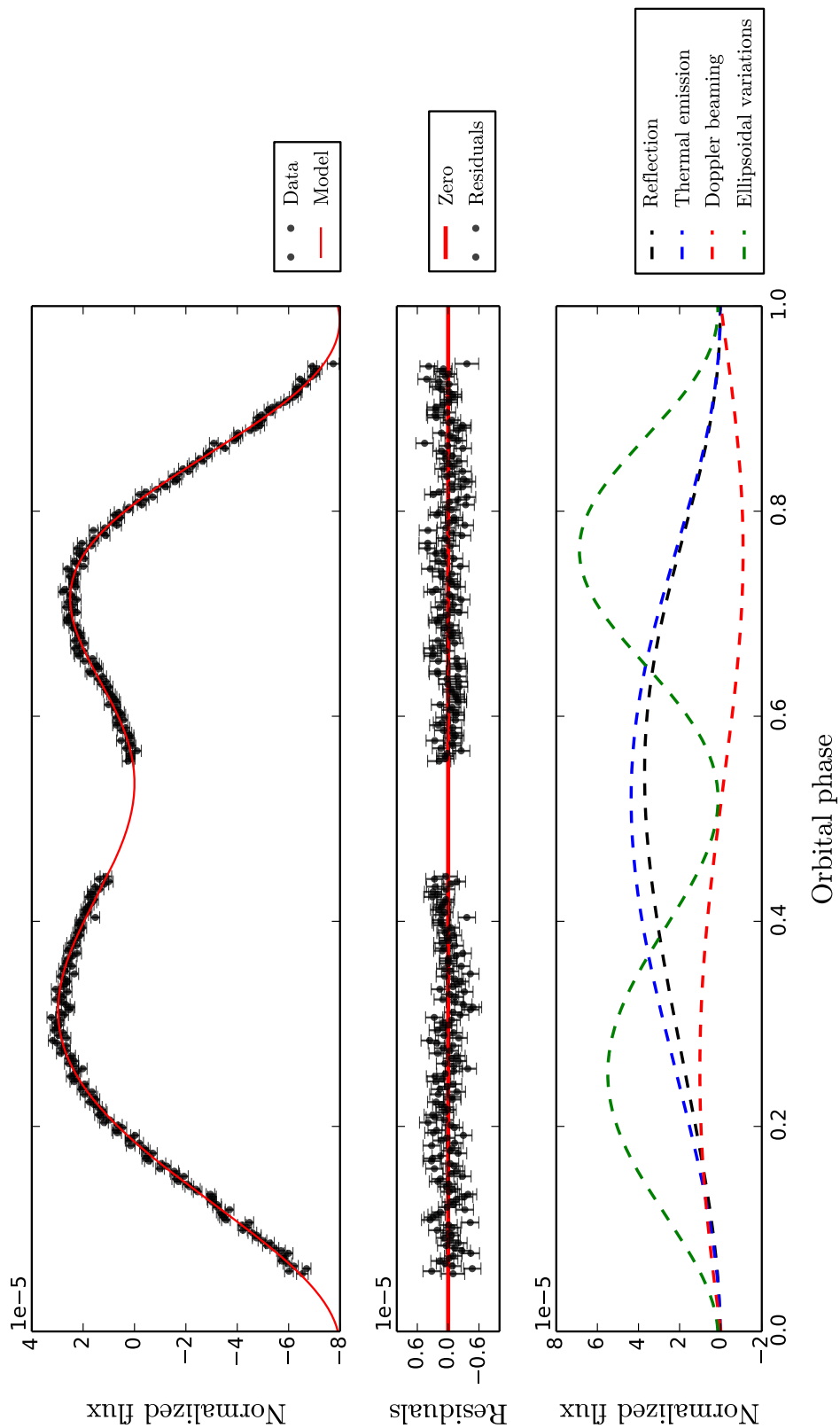


Figure 4.2: Top panel: Best-fitting Re+Th+El+Do model overlaid on Kepler-13b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

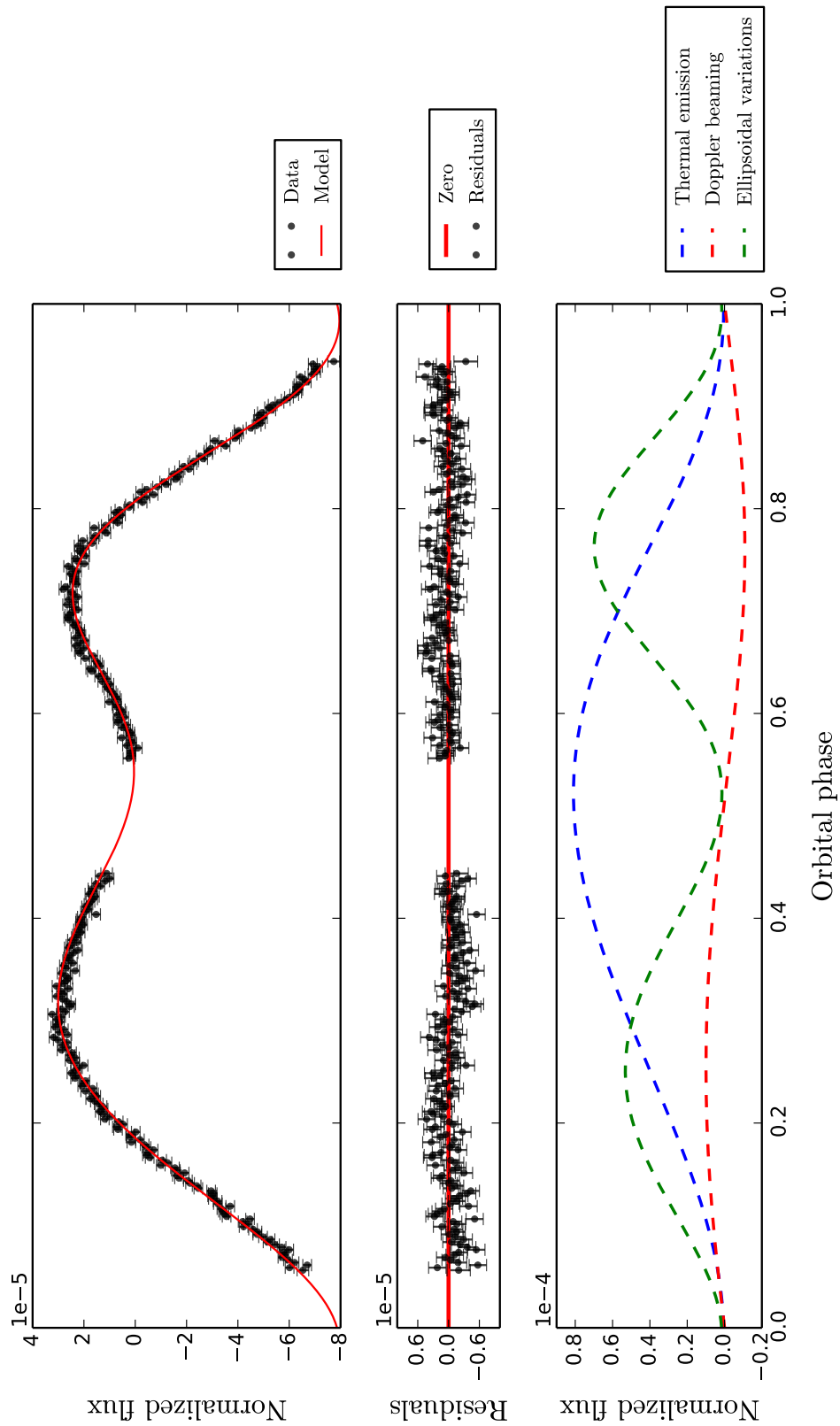


Figure 4.3: Top panel: Best-fitting Th+El+Do model overlaid on Kepler-13b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

4.3 HAT-P-7b

Our fitting of HAT-P-7b (Kepler-2b) does not correspond to literature values as well as our fit for Kepler-13b. Again, we “fixed” the inclination and planetary radius to within 3σ of the published values from transit analyses (see Table 4.6); however, the fitted values of the favored model (Re+El+Do) do not agree well with published values (see Tables 4.8, 4.9). For instance, the planetary mass has been inflated to over twice those of the published values. Figure 4.4 shows the best-fitting model, which despite its unrealistic values, shows good agreement with the data. We re-run our formalism on HAT-P-7b, but narrow the planetary mass bounds to within 3σ of the published radial-velocity/transit mass (see Table 4.6). The triangle plot for the best-fitting Re+El+Do model for these new bounds is shown in Figure 4.5. While the PD is well constrained, the fitting routine compensated for a lower, fixed planetary mass by increasing the eccentricity to over 0.1, which is unrealistic in the case of HAT-P-7b. The fit also increased the effective radius, which places our fitted albedo even further outside the range given by Esteves et al. (2013).

We run the HAT-P-7b data a third time, now forcing the eccentricity to take values between 0 and 0.05, as we observed no asymmetry in the temporal separation of transit and occultation when reducing the data. Figure 4.7 shows the triangle plot for the best-fitting Re+El+Do run in this limited eccentricity case, which demonstrates the walkers building up against the upper e bound of 0.05 in an attempt to unrealistically increase the eccentricity. Simultaneously, the effective radius increases, deviating further from the results of Esteves et al. (2013). The corresponding plots of the best-fitting models are given in Figures 4.6 and 4.8, and show rather poor correspondence to the data.

Parameter	Published value	Reference
$M_{\star} [M_{\odot}]$	1.47	Pál et al. 2008
$R_{\star} [R_{\odot}]$	1.84	Pál et al. 2008
$T_{\text{eff}} [\text{K}]$	6350	Pál et al. 2008
$P [\text{days}]$	2.204737	Morris et al. 2013
$i [\text{deg}]$	83.111 ± 0.030	Morris et al. 2013
$R_{\text{p}}/R_{\star}$	0.07759 ± 0.00003	Morris et al. 2013
$M_{\text{p}} [M_{\text{J}}]$	1.776 ± 0.077	Pál et al. 2008

Table 4.6: Same as Table 4.1, but for HAT-P-7b.

Model	χ^2_{ν}	Model	χ^2_{ν}
Re+El+Do	2.7745	Re+El+Do	2.7745
Re+Th+El+Do	2.7931	Re+El	3.2735
Th+El+Do	2.8862	Re	3.3825
Re+El	3.2735	Re+Do	3.3792
Re+Th+El	3.2958	Th+El+Do	2.8862
Re+Do	3.3792	Re+Th+El+Do	2.7931
Re	3.3825	Re+Th+El	3.2958
Re+Th+Do	3.4014	Re+Th+Do	3.4014
Th+El	4.1303	Th	4.2651
Th	4.2651	Th+El	4.1303
Th+Do	4.2793	Th+Do	4.2793
Re+Th	4.4890	Re+Th	4.4890
Do	6.2015	Do	6.2015
El	19.9684	El	19.9684
El+Do	28.7026	El+Do	28.7026
null	110.4063	null	110.4063

(a)

(b)

Table 4.7: Same as Table 4.3, but for HAT-P-7b.

Parameter	Best	Median
<i>Re+El+Do</i>		
e	0.0320	$0.0318^{+0.0041}_{-0.0037}$
i [deg]	83.092	$83.110^{+0.062}_{-0.060}$
ω [rad]	4.352	$4.353^{+0.091}_{-0.111}$
M_0 [rad]	0.276	$0.275^{+0.109}_{-0.088}$
M_p [M _J]	5.62	$5.62^{+0.17}_{-0.17}$
$\sqrt{A_g} R_p$ [R _J]	0.6625	$0.6624^{+0.0066}_{-0.0068}$
A_g	—	$0.227^{+0.057}_{-0.028}$

Table 4.8: Results of the phase two runs for HAT-P-7b for the most statistically significant model. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit. Since the Re+El+Do model fits only the effective radius, A_g was calculated using the fitted $\sqrt{A_g} R_p$ and published R_p values.

Parameter	Placek & Knuth 2015	Esteves et al. 2013
M_0 [rad]	4.9705 ± 0.0001	—
M_p [M _J]	1.66 ± 0.16	1.985 ± 0.070
A_g	0.088 ± 0.026	0.0299 ± 0.0041
T_{day} [K]	2859.0 ± 33.0	2784 ± 35

Table 4.9: Same as Table 4.5, but for HAT-P-7b.

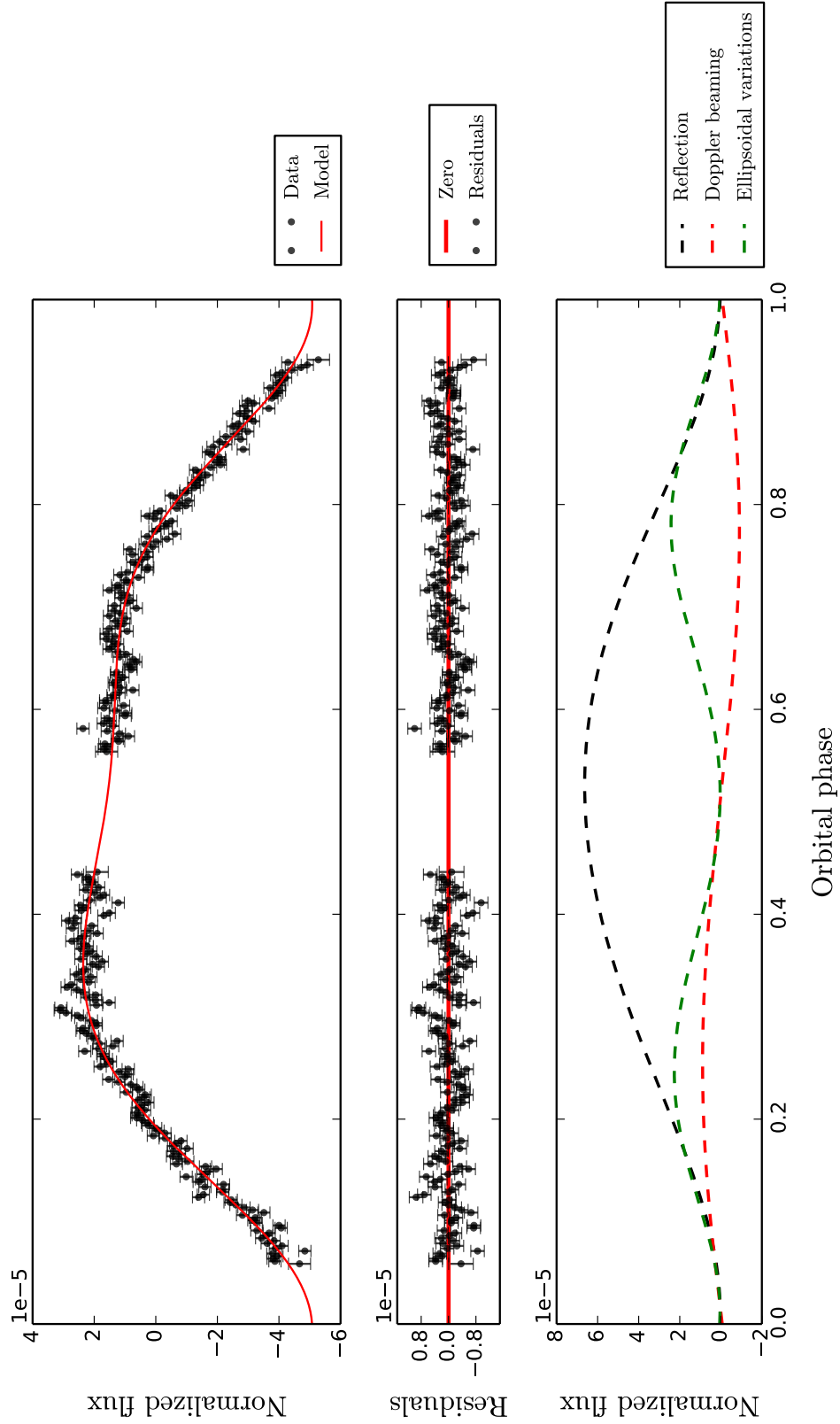


Figure 4.4: Top panel: Best-fitting Re+El+Do model overlaid on HAT-P-7b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

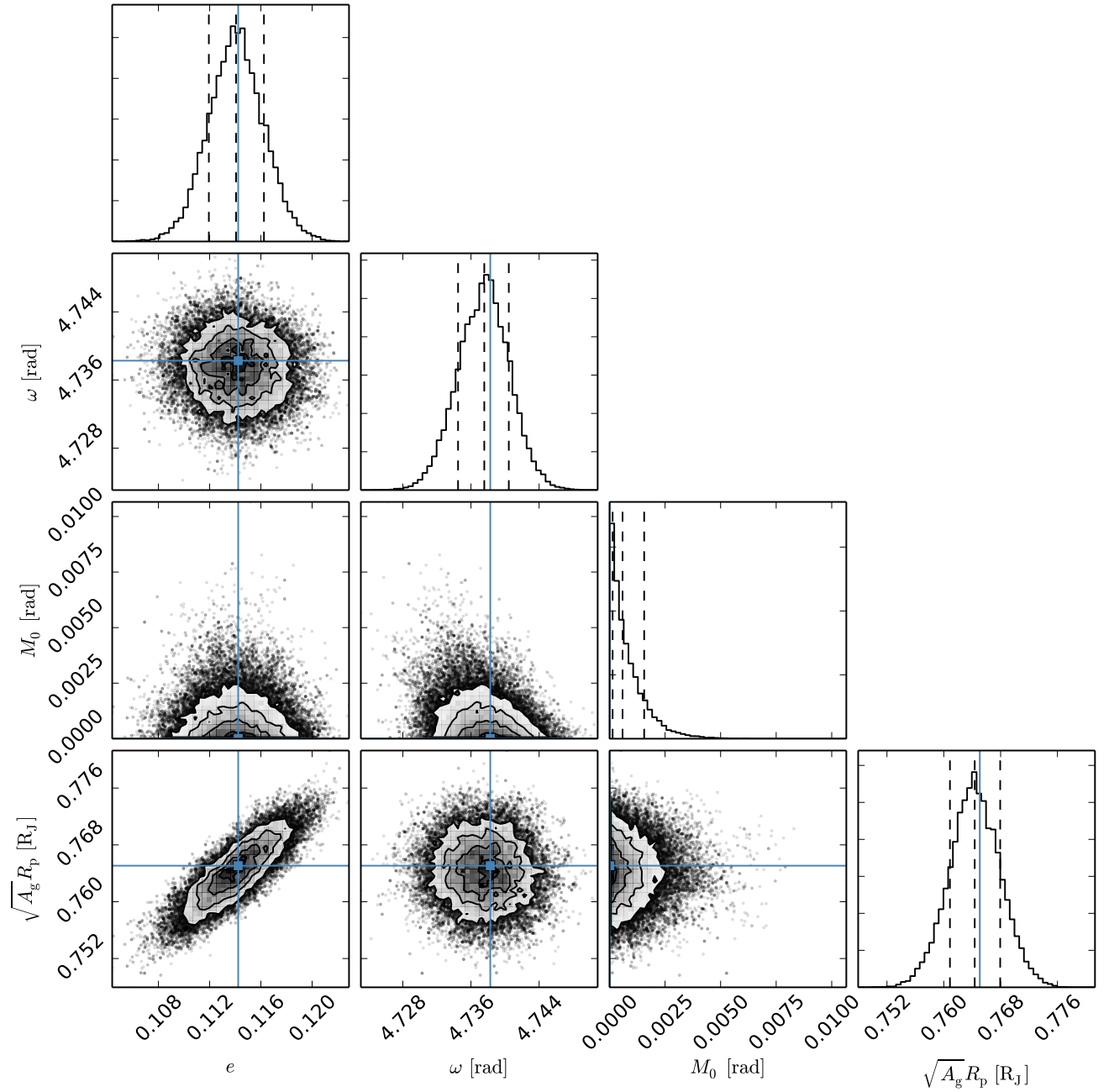


Figure 4.5: Triangle plot of the phase two run (fixed M_p) that yielded the best fit of the Re+El+Do model for HAT-P-7b.

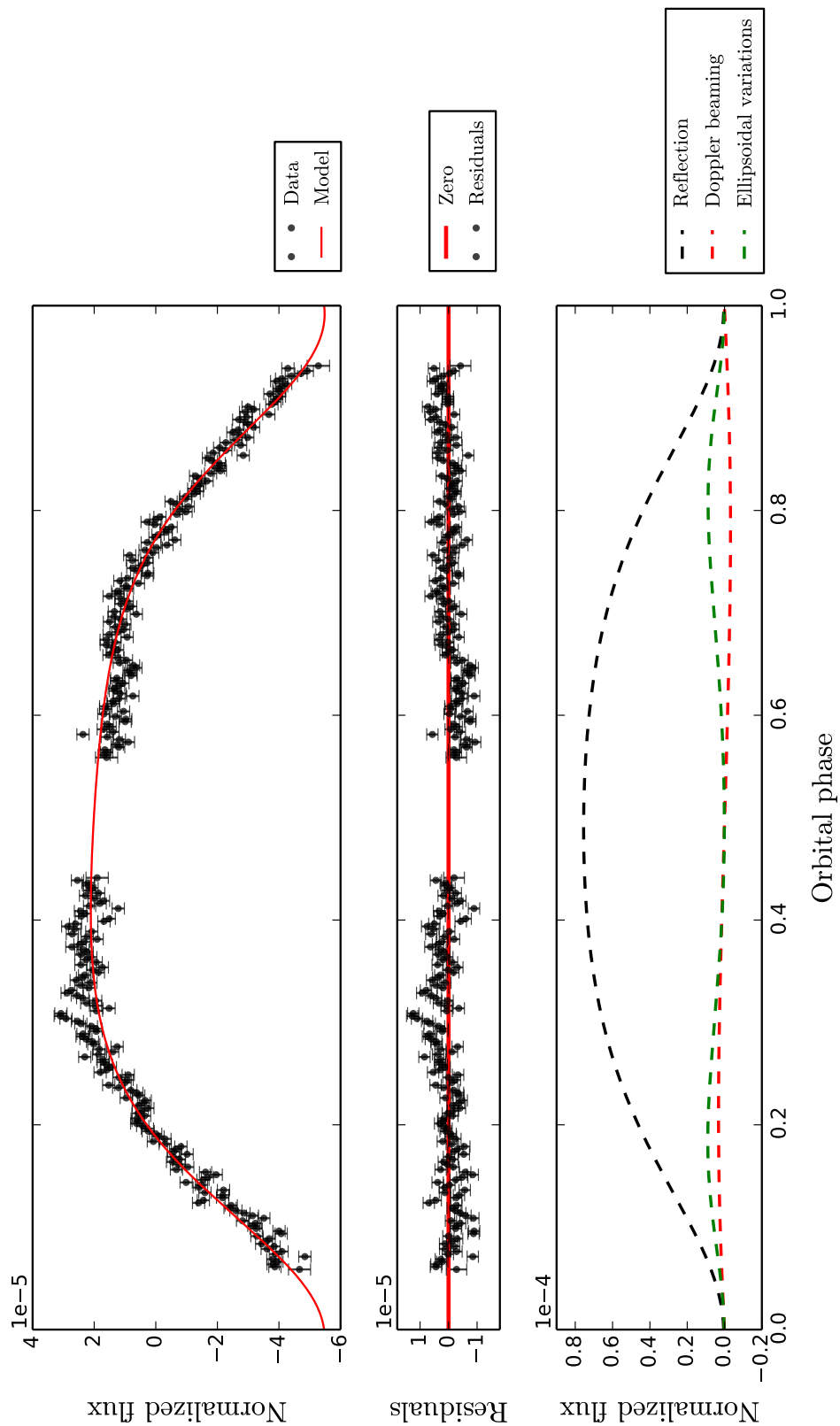


Figure 4.6: Top panel: Best-fitting Re+El+Do model (fixed M_p) overlaid on HAT-P-7b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

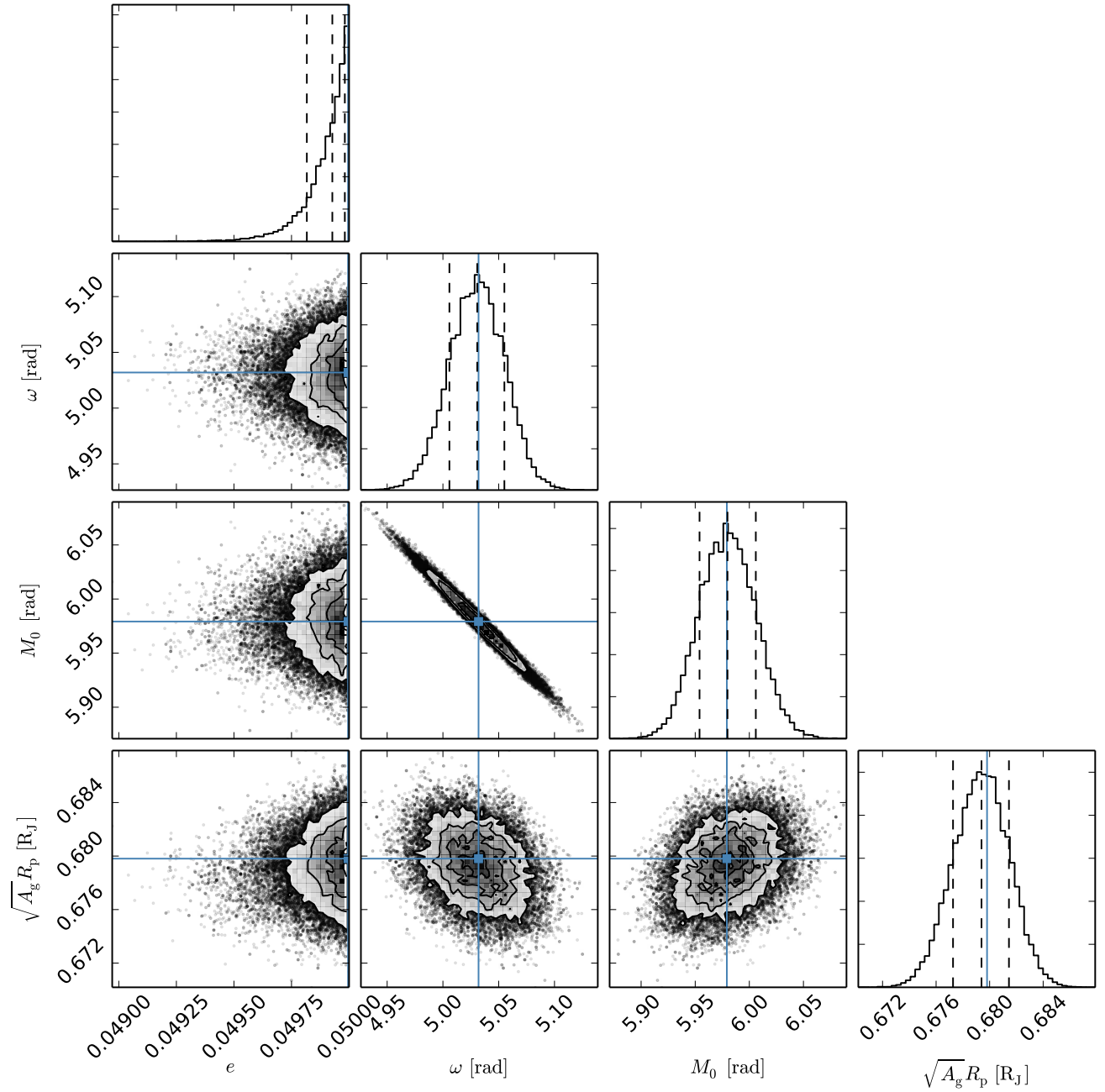


Figure 4.7: Triangle plot of the phase two run (fixed M_p , e constrained from 0 to 0.05) that yielded the best fit of the Re+El+Do model for HAT-P-7b.

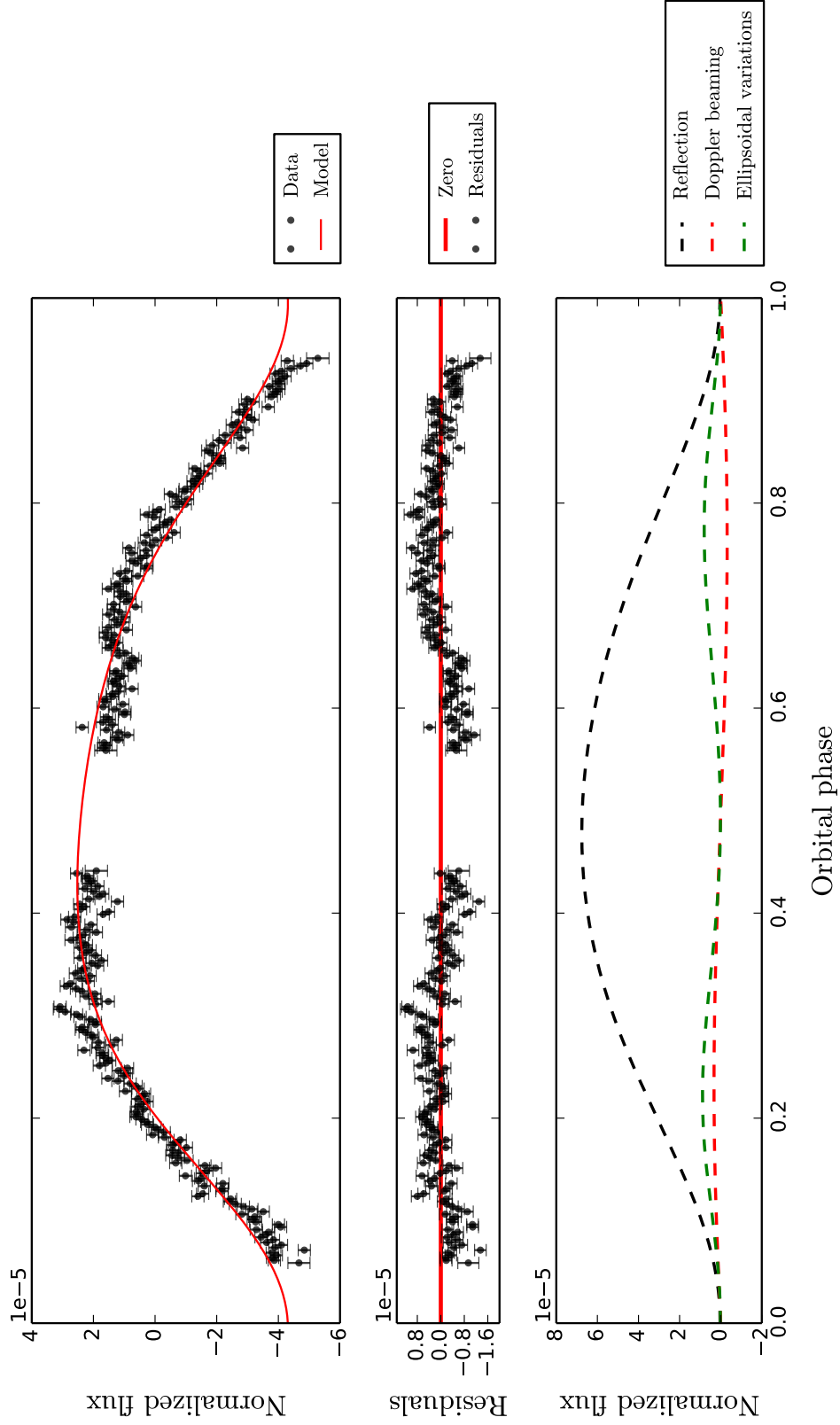


Figure 4.8: Top panel: Best-fitting Re+El+Do model (fixed M_p , eccentricity limited from 0 to 0.05) overlaid on HAT-P-7b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

4.4 Kepler-5b

Fitting Kepler-5b with inclination and radius fixed (see Table 4.10) resulted in a null detection; that is, the most statistically significant model was the null model, consisting only of a variable flux offset. Such a result is reasonable when one considers that the spread in χ^2_ν values in Table 4.11 is less than two tenths. Nonetheless, the null detection is disconcerting, considering the phase curve has previously been detected and analyzed (Esteves et al. 2013).

Parameter	Published value	Reference
$M_\star$ [$M_\odot$]	1.374	Koch et al. 2010
$R_\star$ [$R_\odot$]	1.793	Koch et al. 2010
T_{eff} [K]	6297	Koch et al. 2010
P [days]	3.548460	Koch et al. 2010
i [deg]	86.3 ± 0.5	Koch et al. 2010
$R_p/R_\star$	$0.08195^{+0.00030}_{-0.00047}$	Koch et al. 2010
M_p [M_J]	$2.114^{+0.056}_{-0.059}$	Koch et al. 2010

Table 4.10: Same as Table 4.1, but for Kepler-5b.

Irregardless of our formalism’s non-detection, we run phase two on the next most significant model, as well as the Re+Th+El+Do model, the results of which are given in Table 4.12. The albedos for both models, as well as the dayside temperature for the four-effect model, agree to within 1σ with those given by Esteves et al. (2013). Our value for planetary mass also agrees well with published radial velocity values (see Table 4.10). However, the eccentricity, with a median value of 0.392, is clearly unrealistic. Figures 4.9 and 4.10 shows the best-fitting models, and offer insight into this inflated eccentricity. Around orbital phase $\phi \approx 0.1$, a dip occurs in the data. To create such an asymmetry, the fitting formalism increases the eccentricity.

We re-run our formalism, but limit the e to take values between 0 and 0.05. The median albedo value from the Re model essentially halves, while the albedo value given by the Re+Th+El+Do model increases by a factor of over 1.5. The T_{day} value given by the Re+Th+El+Do model also more than halves. However, all these values still agree to within $1 - 2\sigma$ of the published Esteves et al. (2013) values, due in part to the their relatively large error bars (see Table 4.14). The planetary mass also agrees well with the published RV value, in spite of the new fitting bounds, which prevent the formalism from fully fitting the dip (see Figures 4.11 and 4.12).

Model	χ^2_ν	Model	χ^2_ν
Th+El+Do	1.1259	null	1.2843
Re+Do	1.1286	Re	1.1485
Re+Th+El+Do	1.1302	Do	1.1583
Re+Th+Do	1.1355	El	1.1740
Re+Th	1.1428	El+Do	1.1852
Re+Th+El	1.1454	Re+Do	1.1286
Re	1.1485	Re+El+Do	1.1518
Th+El	1.1498	Re+El	1.1521
Re+El+Do	1.1518	Th	1.1620
Re+El	1.1521	Th+El+Do	1.1259
Do	1.1583	Re+Th	1.1428
Th	1.1620	Th+El	1.1498
Th+Do	1.1654	Th+Do	1.1654
El	1.1740	Re+Th+El+Do	1.1302
El+Do	1.1852	Re+Th+Do	1.1355
null	1.2843	Re+Th+El	1.1454

(a)

(b)

Table 4.11: Same as Table 4.3, but for Kepler-5b.

Parameter	Best	Median
<i>Re</i>		
e	0.396	$0.387^{+0.031}_{-0.039}$
i [deg]	86.9	$86.3^{+1.0}_{-1.0}$
ω [rad]	5.01	$5.08^{+0.19}_{-0.13}$
M_0 [rad]	5.077	$5.047^{+0.081}_{-0.097}$
$\sqrt{A_g} R_p$ [R _J]	0.495	$0.464^{+0.047}_{-0.054}$
A_g	—	$0.105^{+0.022}_{-0.025}$
<i>Re+Th+El+Do</i>		
e	0.392	$0.392^{+0.047}_{-0.045}$
i [deg]	85.52	$86.37^{+0.98}_{-1.09}$
ω [rad]	4.55	$4.66^{+0.25}_{-0.19}$
M_0 [rad]	5.23	$5.19^{+0.10}_{-0.12}$
M_p [M _J]	3.61	$2.83^{+0.97}_{-1.39}$
R_p [R _J]	1.442	$1.424^{+0.015}_{-0.013}$
A_g	0.002	$0.028^{+0.034}_{-0.020}$
T_{day} [K]	2307	2239^{+73}_{-104}

Table 4.12: Results of the phase two runs for Kepler-5b for the most statistically significant model, as well as the four-effect model. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit. Since the Re model fits only the effective radius, A_g was calculated using the fitted $\sqrt{A_g} R_p$ and published R_p values.

Parameter	Esteves et al. (2013)
M_p [M _J]	$1.34^{+0.30}_{-0.31}$
A_g	$0.065^{+0.032}_{-0.031}$
T_{day} [K]	2198^{+36}_{-35}

Table 4.13: Same as Table 4.5 for Kepler-5b.

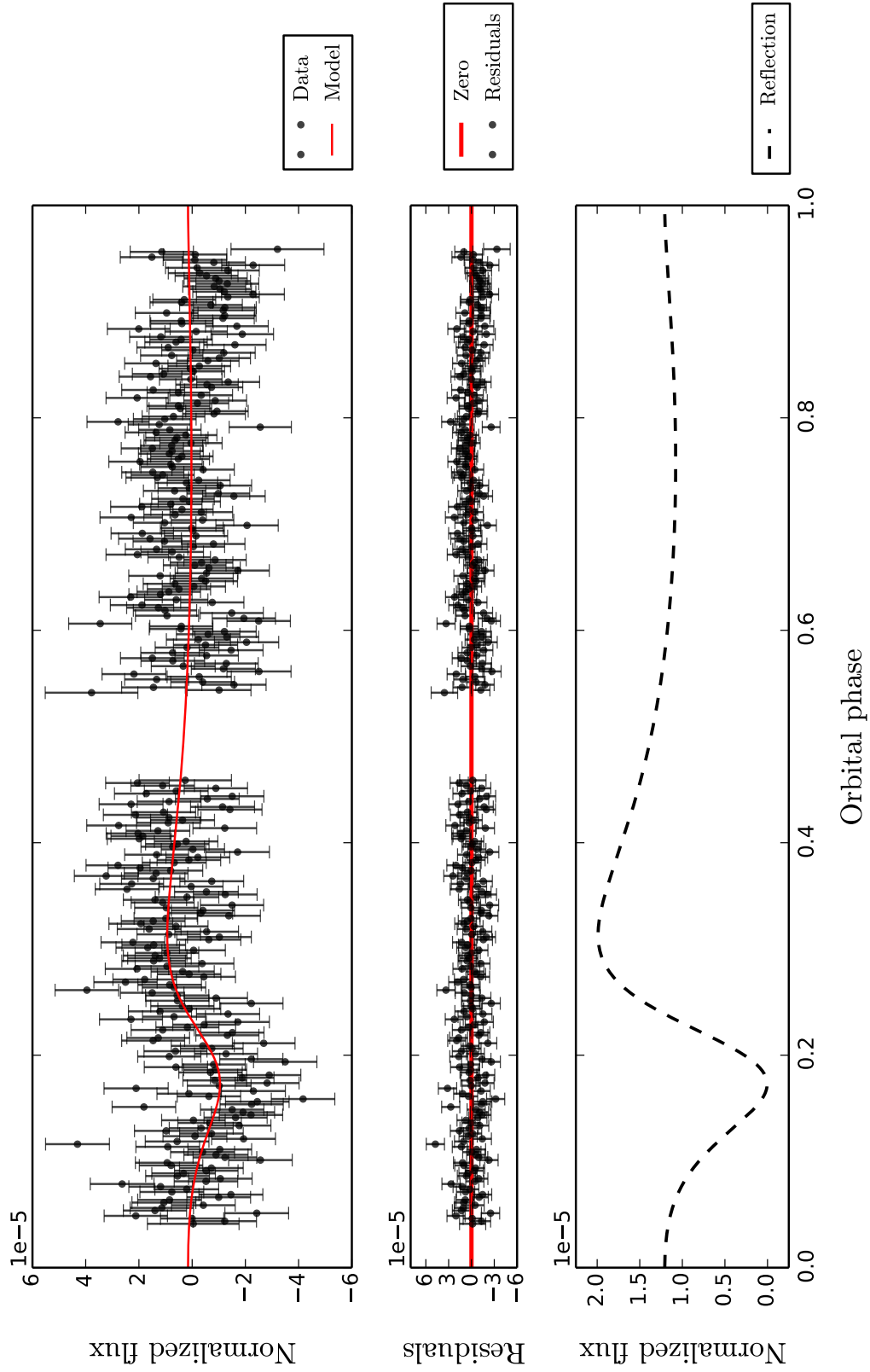


Figure 4.9: Top panel: Best-fitting Re model overlaid on Kepler-5b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

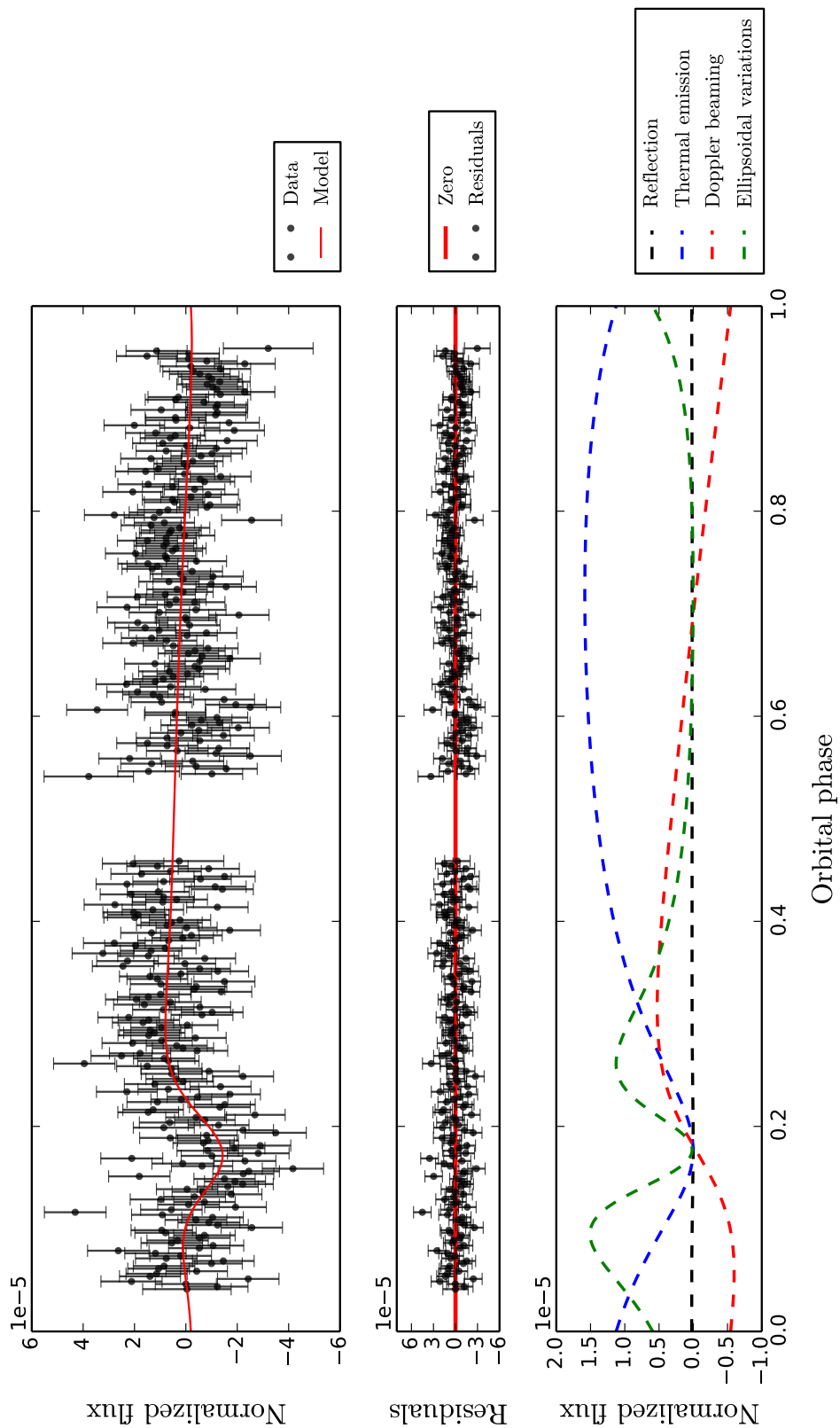


Figure 4.10: Top panel: Best-fitting Re+Th+El+Do model overlaid on Kepler-5b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

Parameter	Best	Median
<i>Re</i>		
e	0.050	$0.033^{+0.012}_{-0.018}$
i [deg]	86.94	$86.27^{+0.99}_{-1.02}$
ω [rad]	5.82	$5.46^{+0.54}_{-0.77}$
M_0 [rad]	5.02	$5.22^{+0.65}_{-0.59}$
$\sqrt{A_g} R_p$ [R _J]	0.347	$0.333^{+0.036}_{-0.042}$
A_g	—	$0.054^{+0.012}_{-0.014}$
<i>Re+Th+El+Do</i>		
e	0.041	$0.031^{+0.014}_{-0.018}$
i [deg]	86.9	$86.3^{+1.0}_{-1.0}$
ω [rad]	5.90	$5.41^{+0.57}_{-0.80}$
M_0 [rad]	4.34	$4.90^{+0.78}_{-0.58}$
M_p [M _J]	2.40	$2.28^{+0.61}_{-0.64}$
R_p [R _J]	1.408	$1.425^{+0.014}_{-0.013}$
A_g	0.017	$0.044^{+0.013}_{-0.013}$
T_{day} [K]	2020	940^{+710}_{-650}

Table 4.14: Results of the phase two runs (e limited from 0 to 0.05) for Kepler-5b for the most statistically significant model, as well as the four-effect model. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit. Since the Re model fits only the effective radius, A_g was calculated using the fitted $\sqrt{A_g} R_p$ and published R_p values.

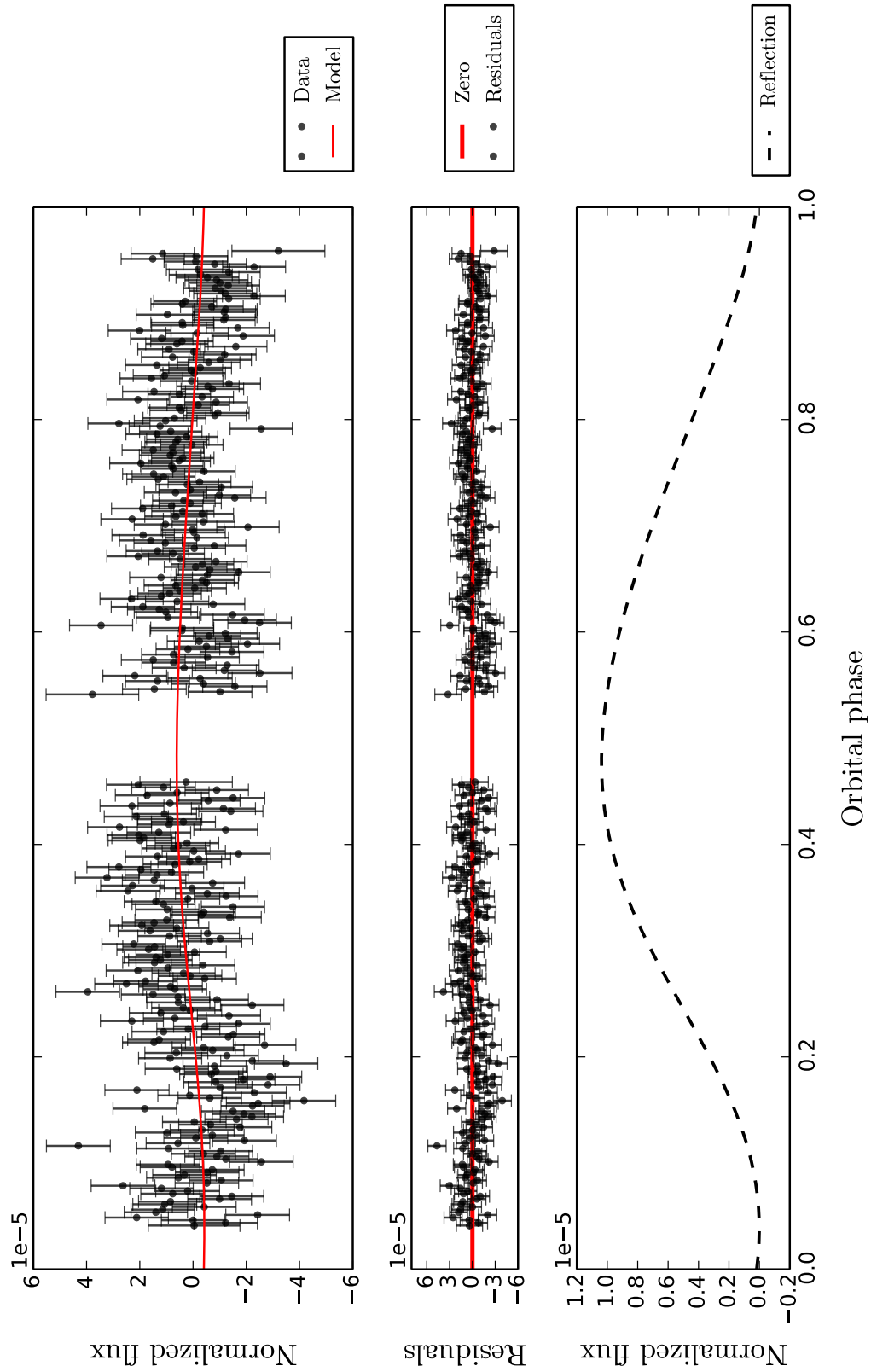


Figure 4.11: Top panel: Best-fitting Re model (e limited from 0 to 0.05) overlaid on Kepler-5b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

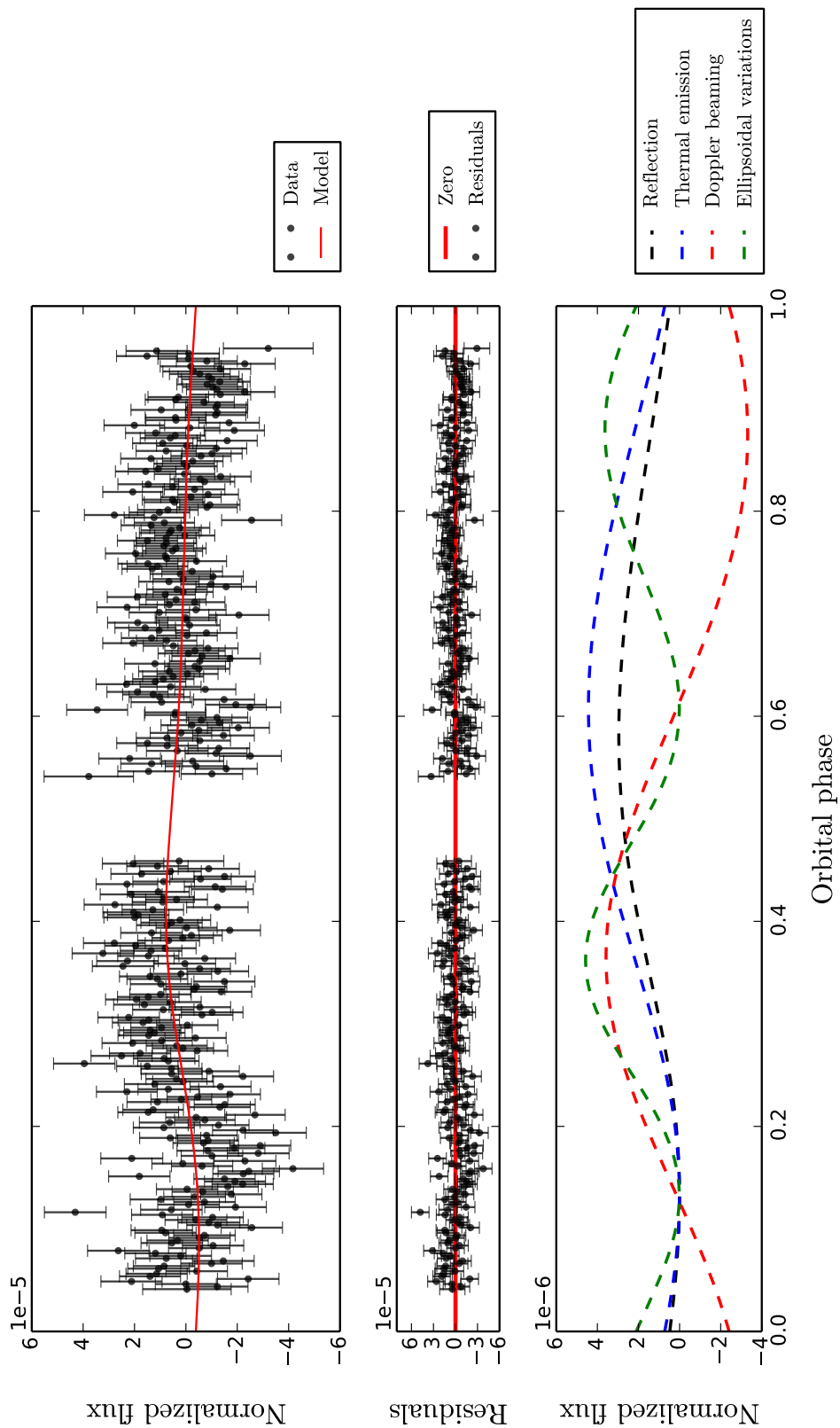


Figure 4.12: Top panel: Best-fitting Re+Th+El+Do model (e limited from 0 to 0.05) overlaid on Kepler-5b data. Middle panel: Residuals. Bottom panel: Constituent phase effects.

4.5 Kepler-6b and Kepler-8b

Our fits for Kepler-6b and Kepler-8b demonstrate problems similar to those experienced by HAT-P-7b and Kepler-5b. We therefore present a limited summary of the results to avoid redundancy. Tables 4.15 and 4.16 give the published stellar parameters and transit-derived parameters we use in our fitting, as well as the radial velocity-determined mass for comparison. Similar to Kepler-5b, both Kepler-6b and Kepler-8b were null detections. We present results of phase two of our fitting formalism for the four-effect model in Tables 4.17 and 4.18, for which we fixed the inclination and planetary radius as in previous cases. For Kepler-6b, the median values for both eccentricity and planetary mass are inflated, while only the eccentricity is inflated in the case of Kepler-8b. In both cases, A_g and T_{day} agree to within 1σ to those values given by Esteves et al. (2013) (see Tables 4.19 and 4.20).

Parameter	Published value	Reference
$M_\star [M_\odot]$	1.209	Dunham et al. 2010
$R_\star [R_\odot]$	1.391	Dunham et al. 2010
$T_{\text{eff}} [\text{K}]$	5647	Dunham et al. 2010
$P [\text{days}]$	3.234723	Dunham et al. 2010
$i [\text{deg}]$	86.8 ± 0.3	Dunham et al. 2010
$R_p/R_\star$	$0.09829^{+0.00014}_{-0.00050}$	Dunham et al. 2010
$M_p [M_J]$	$0.669^{+0.025}_{-0.030}$	Dunham et al. 2010

Table 4.15: Same as Table 4.1, but for Kepler-6b.

Parameter	Published value	Reference
$M_\star [M_\odot]$	1.213	Jenkins et al. 2010
$R_\star [R_\odot]$	1.486	Jenkins et al. 2010
$T_{\text{eff}} [\text{K}]$	6213	Jenkins et al. 2010
$P [\text{days}]$	3.52254	Jenkins et al. 2010
$i [\text{deg}]$	84.07 ± 0.33	Jenkins et al. 2010
$R_p/R_\star$	$0.09809^{+0.00040}_{-0.00046}$	Jenkins et al. 2010
$M_p [M_J]$	$0.603^{+0.13}_{-0.19}$	Jenkins et al. 2010

Table 4.16: Same as Table 4.1, but for Kepler-8b.

Parameter	Best	Median
<i>Re+Th+El+Do</i>		
e	0.259	$0.210^{+0.061}_{-0.075}$
$i [\text{deg}]$	86.88	$86.83^{+0.61}_{-0.64}$
$\omega [\text{rad}]$	3.45	$3.44^{+0.48}_{-0.53}$
$M_0 [\text{rad}]$	0.62	$0.62^{+0.39}_{-0.33}$
$M_p [M_J]$	5.3	$5.0^{+1.1}_{-1.3}$
$R_p [R_J]$	1.3143	$1.3245^{+0.0088}_{-0.0101}$
A_g	0.000	$0.018^{+0.025}_{-0.013}$
$T_{\text{day}} [\text{K}]$	2120	2010^{+130}_{-240}

Table 4.17: Results of the phase two runs for Kepler-6b for the four-effect model. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit.

Parameter	Best	Median
<i>Re+Th+El+Do</i>		
e	0.29	$0.22^{+0.12}_{-0.10}$
i [deg]	83.21	$84.09^{+0.68}_{-0.69}$
ω [rad]	2.61	$2.65^{+0.61}_{-0.48}$
M_0 [rad]	1.88	$1.82^{+0.27}_{-0.44}$
M_p [M _J]	1.48	$1.42^{+1.09}_{-0.85}$
R_p [R _J]	1.412	$1.417^{+0.013}_{-0.013}$
A_g	0.039	$0.058^{+0.029}_{-0.030}$
T_{day} [K]	2010	1680^{+390}_{-850}

Table 4.18: Results of the phase two runs for Kepler-8b for the four-effect model. The second column gives the parameter values corresponding to the best- χ^2_ν fit over the ten runs. The third column gives the median parameter values with 1σ errors for the best- χ^2_ν fit.

Parameter	Esteves et al. (2013)
M_p [M _J]	1.02 ± 0.40
A_g	0.038 ± 0.028
T_{day} [K]	1829 ± 25

Table 4.19: Same as Table 4.5, but for Kepler-6b.

Parameter	Esteves et al. (2013)
M_p [M _J]	1.35 ± 0.39
A_g	0.098 ± 0.036
T_{day} [K]	2066 ± 60

Table 4.20: Same as Table 4.5, but for Kepler-8b.

4.6 Analysis

Our analyses of Kepler-13b, HAT-P-7b, and Kepler-5b reveal several interesting issues with out fitting formalism. Perhaps the most significant is the null detection of a star system with a known planetary light curve (i.e., Kepler-5b). Multiple factors may influence this outcome. For instance, it is possible our data reduction detrends too far, decreasing the amplitudes of the astrophysical phase curve signals. The published detrended light curve of Kepler-5b is certainly less pronounced than either of the other two planets we tested (cf. Placek et al. 2014, Placek & Knuth 2015, Esteves et al. 2013). Perhaps the spline method we used had a greater impact on Kepler-5b, which resulted in any variations having amplitudes on order of the noise values. We may also try another statistical test for comparing models' goodness of fits. The F-test may simply be too stringent in its application of Occam's razor.

Another interesting problem we encounter is the inflation of the eccentricity, again, for Kepler-5b. As previously mentioned in the eponymous section, we attribute the inflation in eccentricity to the sharp dip in the reduced light curve of Kepler-5b. A similarly obvious deviation is not apparent in the corresponding light curve from Esteves et al. (2013) (see Figure 4.13). This lends credence to the notion that the data reduction is culpable for these fitting issues, however, it is worth noting that all of the other fitted parameters agree with those published by Esteves et al. (2013).

The underlying cause of the planetary mass inflation in the case of HAT-P-7b, followed by the e inflation when the mass was “fixed” is less obvious. Comparing the plots of the best-fitting Re+El+Do models for that planet, it is clear that the asymmetric bimodality of the data is well replicated in Figure 4.4, where the

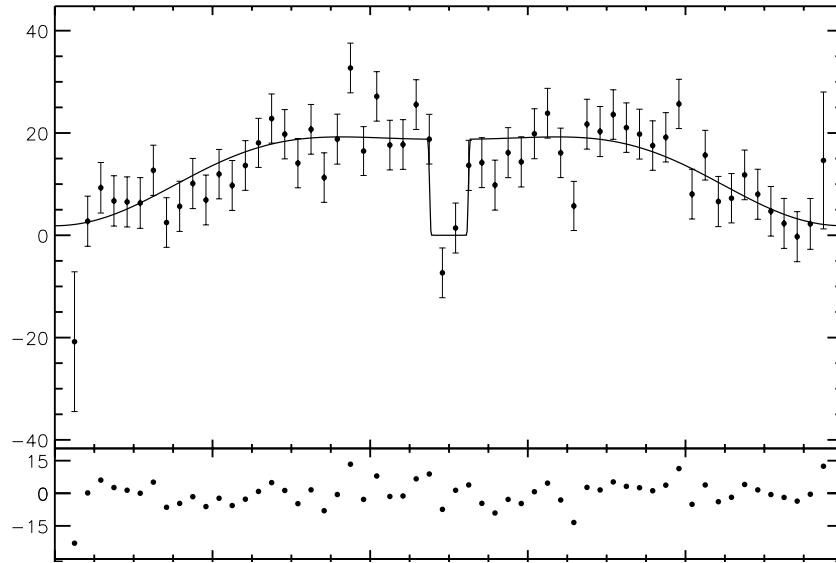


Figure 4.13: Data and best-fitting model for Kepler-5b from Esteves et al. (2013).

confluence of ellipsoidal variations and Doppler beaming causes one peak to be stronger than the other. Fixing the mass at the lesser, true value (see Figure 4.6) forces the ellipsoidal and Doppler amplitudes to lessen, causing less of an asymmetry in the strengths of the phase curve peaks. The fitting routine seems to attempt to compensate for this by increasing the eccentricity, thereby flattening the peak of the reflection curve. This cannot adequately reproduce the asymmetry, however, and both the graph of model overplotted on the data, and the graph of the residuals, indicate a poor correspondence. Figure 4.8 further demonstrates this. The fitting routine increases the eccentricity as far as the bounds allow, which is now severely restricted. Consequently, the reflection curve is even more unimodal, and the fit poorer.

Perhaps the data reduction also causes the mass inflation; however, comparing our reduced data for HAT-P-7b with that of Esteves et al. (2013) indicates that apart from a different amount of binning, the overall phase curve shapes and amplitudes are rather similar (see Figure 4.14). This indicates the data reduc-

tion cannot be the primary cause of the planetary mass/eccentricity inflation for HAT-P-7b. As the amplitudes of the ellipsoidal variations feature in this fitting dilemma, another possibility is that the gravity-darkening β term that contributes to the ellipsoidal amplitude is improperly calculated. Our rather simple method for calculating this leading coefficient may not be adequate; for instance, Esteves et al. (2013) use both gravity- and limb-darkening terms in their determination of the ellipsoidal amplitude. On the other hand, the method used by Placek et al. (2014), upon which we base our calculation, accurately fits the planetary mass for HAT-P-7b (Placek & Knuth 2015). Clearly then, further investigation is required to determine how to improve the accuracy of our fitting results.

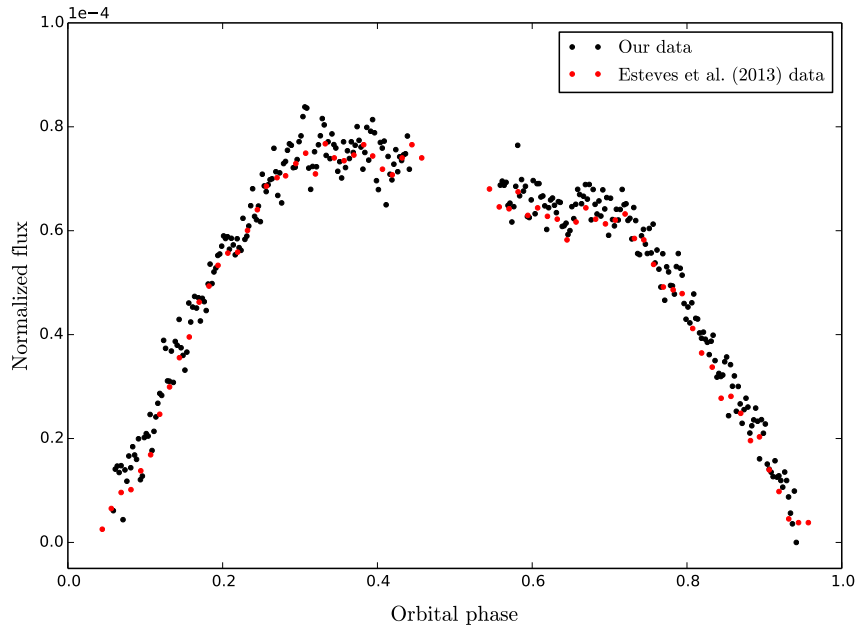


Figure 4.14: Our reduced data for HAT-P-7b (black) overplotted with that of Esteves et al. (2013) (red).

Chapter 5

Conclusion

Astronomy has progressed substantially from the time of Albertus Magnus, to the point that we can now decisively assert there are in fact many worlds, both within and beyond our solar system. We have shown it is feasible to characterize these exoplanets by analyzing their phase curves, the small scale variations in a star-exoplanet system’s light curve that occur as the exoplanet orbits. This in itself is a rather impressive development, as the precision necessary to do so has only recently become available for large numbers of stars with the advent of Kepler.

While this method holds great potential, we encountered several limitations that must be considered. One of the most significant are the major degeneracies between different models, as well as individual parameters. We have shown that for certain planet’s light curves, our fitting formalism will prefer a single-component model that grossly exaggerates certain parameters. We have also seen certain parameters—namely, R_p , A_g , and T_{day} —become degenerate, where a data set can be equally well fit by altering any one of the three. By using prior knowledge from other exoplanet characterization techniques to fix, or at least narrowly constrain, certain parameters, we can somewhat mitigate these issues; however, degeneracy still remains, and the number of truly free parameters in our fit decreases.

We also suspect data reduction routines introduce artifacts into the light curves, or detrend them too far, thereby removing astrophysical signals. We may need to reevaluate our expectations of what constitutes acceptable spacecraft systematics, in light of the introduction of significant reduction systematics. A wider spread in the light curve courtesy of Kepler may indeed be preferable to severe fluctuations in the data due to our reduction routine.

Despite these limitations, we still see phase curve analysis as a viable method to characterize exoplanets. In addition to minimizing the impact of the aforementioned limitations, there are other improvements we can make to our analysis technique. We also foresee specific applications that have the potential to significantly impact the field of exoplanets.

5.1 Future work

5.1.1 Incorporating transits and occultations

Our phase curve model currently only considers the out-of-transit and out-of-occultation phase variations. In our fitting, therefore, we included information from previous transit fits by narrowing the bounds of the fitting routine. A more robust and self-consistent method for including transit data is to actually fit the transit simultaneously with the phase curves. A transit fit would easily determine i and R_p . If the light curve also contains an occultation, the separation between these two events determines very accurately the eccentricity of the orbit. Expanding the light curve model would thus determine very well several parameters we have tried to constrain in a rather ad hoc method.

Fitting the occultation would also provide further constraints on the reflection and thermal effects. Specifically, the depth of the occultation is equal to the

combined reflected and thermal amplitudes at full phase. While this will not assist in disentangling the two effects, it will resolve any degeneracies between reflection/thermal and ellipsoidal/beaming components.

5.1.2 Non-transiting planets

As alluded to in Chapter 1, a rather exciting prospect for phase curve analysis is its potential to detect non-transiting exoplanets, thereby allowing combined radial velocity/photometric studies of the vast majority of planets in the Universe. For these cases, we will need a quick, efficient method to determine from a light curve whether there is a planetary signal. An obvious choice would be some sort of periodogram analysis, since a phase curve—albeit a weak effect compared to a transit—should produce a peak at the planet’s orbital period. Figure 5.1 is a basic comparison between the periodogram signals of a simple box transit model of an exoplanet similar to Kepler-13b and the phase curve variations of that same planet inclined such that it no longer transits. As expected, the raw value of the maximum signal for the non-transiting planet is weaker than that of the transiting planet, in this case, by over a factor of 400. The aliasing seems to be less pronounced for the non-transiting case, for which we have yet to develop an explanation. In the near future, we plan to further develop such a method for detecting candidate non-transiting planets for followup phase curve analysis.

5.1.3 Future targets: K1 and K2

As the K1 mission targeted over 100,000 stars, there is a bounty of light curves through which to search for phase curves. Rather than simply feeding the entire K1 database through our fitting formalism, we envisage a few differ-

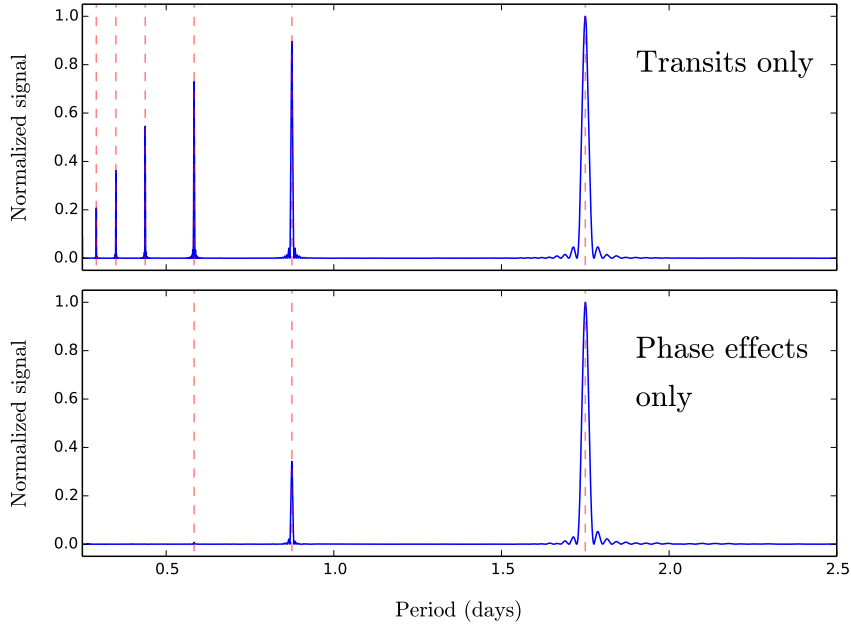


Figure 5.1: Periodograms of a box-model transit (top panel) and phase variations (bottom panel) for a hot Jupiter similar to Kepler-13b that transits edge-on and does not transit, respectively. The signals are normalized to show the relative strengths of the aliasing.

ent, more targeted approaches. Perhaps the most obvious is to apply a future phase curve/transit/occultation fitting algorithm to all known Kepler transiting exoplanets, with the hope of providing further characterization of already studied systems. Assuming we are able to devise a method to flag potential non-transiting exoplanets, a perhaps more fruitful endeavor would be to analyze the hottest Kepler targets for signs of both transiting and non-transiting exoplanets. Hot targets are particularly intriguing in the case of transiting planets because they are most amenable to followup atmospheric spectroscopic observations, in that the higher stellar flux presents an opportunity to achieve lower signal to noise. For non-transiting planets, hot targets will likely produce phase variations with greater amplitudes, enabling easier detection.

The K2 phase of the Kepler mission will likely continue to take data for the

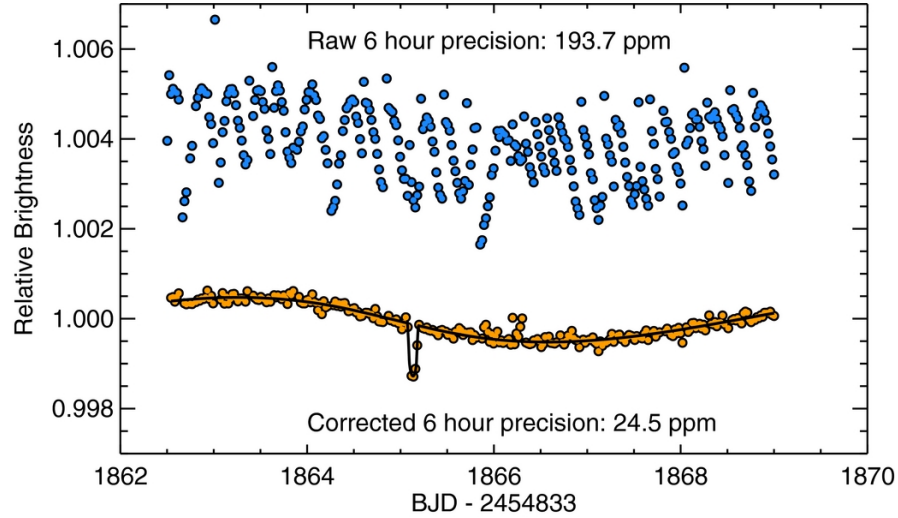


Figure 5.2: K2 light curve of the super-Earth discovered by Vanderburg et al. (2015), which shows out-of-transit phase variations.

next couple of years, until its fuel stores are exhausted. In the meantime, as it traces the ecliptic plane, its overall field of view has expanded beyond the original 115 deg^2 patch of sky selected for K1. This means more opportunities to detect and characterize exoplanets, both from transits and via their phase curves. In fact, the previously mentioned K2 planet discovery made by Vanderburg et al. (2015) shows signs of phase effects (see Figure 5.2). We are hopeful, then, that for the foreseeable future, the archive of precision light curves will continue to grow, and with it, the number of exoplanets with characterizable light curves.

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